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ORIGINAL ARTICLE

Exact solutions of the Zoomeron and Klein–Gordon–Zakharov equations

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Abstract The first integral method was used to construct exact solutions of the Zoomeron and Klein–Gordon–Zakharov equations. The obtained results include new soliton and periodic solutions. The work confirms the significant features of the employed method and shows the variety of the obtained solutions. Throughout the paper, all the calculations are made with the aid of the Maple packet program.

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1. Introduction

In the field of nonlinear science, nonlinear evolution equations (NLEEs) are often presented to describe the motion of isolated waves, localized in a small part of space, in many fields such as hydrodynamics, nonlinear optics, plasma physics, optical fibers, biology, chemical kinematics, solid state physics, chemical physics, etc. Especially, obtaining their explicit solutions is even more difficult. So far, a lot of methods for solving nonlinear differential equations are developed, for example, inverse scattering method, Backlund transformation method, Painleve expansion method, Darboux transformation method, Hirota's bilinear method, and so forth. Besides, generally speaking, all of the above methods have their own advantages and shortcomings, respectively.

In the past several decades, new exact solutions may help to find new phenomena. A variety of powerful methods, such as inverse scattering method (Ablowitz and Clarkson, 1990; Vakhnenko et al., 2003), bilinear transformation Hirota (1980), homogeneous balance method (Fan and Zhang, 1998), the tanh-sech method (Malfliet and Hereman, 1996; Wazwaz, 2004), extended tanh method (Fan, 2000; Wazwaz, 2007), Exp-function method (He and Wu, 2006; Zhang, 2007), sine–cosine method (Wazwaz, 2004; Bekir, 2008) and functional variable method Cevikel et al. (in press) were used to develop nonlinear dispersive and dissipative problems.

In recent years, there have been many works on the qualitative research of the global solutions for the Klein–Gordon–Zakharov equations (Guo and Yuan, 1995; Tsutaya, 1996). Chen Lin considered orbital stability of solitary waves for the Klein–Gordon–Zakharov equations in Chen (1999). More recently, some exact solutions for this equation are obtained by using different methods (Ebadi et al., 2010; Shang et al., 2008). Lately, some exact solutions of the Zoomeron equation have also been found by some authors using extended tanh method, exponential function method, sech–tanh method, tanh–coth method, and $(\frac{G'}{G})$-expansion method (Alquran and

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Al-Khaled, 2012; Irshad and Mohyud-Din, in press; Abazari, 2011). These solutions are not general and by no means exhaust all possibilities. They are only some particular solutions within some specific parameter choices.

The remainder of this paper is organized as follows. In Sections 2–4, using the first-integral method which is based on the ring theory of commutative algebra, we establish the exact solutions for Zoomeron and Klein–Gordon–Zakharov equations, which is in full agreement with the previously known result in the literature. However, our results provide a good supplement to the existing literatures. Finally, some conclusions are given in Section 5.

2. The first integral method

The pioneer work of Feng (2002) introduced the first integral method for a reliable treatment of the nonlinear PDEs. The useful first integral method is widely used by many such as in (Feng and Wang, 2003; Ahmed Ali and Raslan, 2007; Tascan et al., 2009; Moosaei et al., 2011; Jafari et al., 2012) and by the reference therein. Raslan has summarized for using first integral method Raslan (2008).

Step 1. Take a general nonlinear PDE in the form

$$P(u, u_t, u_x, u_{xx}, u_{tt}, u_{xt}, u_{xxx}, \ldots) = 0. \tag{2.1}$$

Employing a wave variable $\xi = x - ct$, we can rewrite Eq. (2.1) as nonlinear ODE

$$Q(U, U', U'', U''', \ldots) = 0, \tag{2.2}$$

where the prime denotes the derivation with respect to ξ. Eq. (2.2) is then integrated as long as all terms contain derivatives where integration constants are considered zeros.

Step 2. We think that the solution of ODE (2.2) can be written in the form:

$$u(x, t) = f(\xi). \tag{2.3}$$

Step 3. We introduce a new independent variable

$$X(\xi) = f(\xi), \quad Y = f_\xi(\xi), \tag{2.4}$$

which leads a system of

$$\begin{aligned} X_\xi(\xi) &= Y(\xi), \\ Y_\xi(\xi) &= F(X(\xi), Y(\xi)). \end{aligned} \tag{2.5}$$

Step 4. By the qualitative theory of ordinary differential equations (Ding and Li, 1996), if we can find the integrals to (2.5) under the same conditions, then the general solutions to (2.5) can be solved directly. Withal, in general, it is really difficult for us to realize this even for one first integral, because for a given plane autonomous system, there is no systematic theory that can tell us how to find its first integrals, nor is there a logical way for telling us what these first integrals are. We will apply the Division Theorem to obtain one first integral to (2.5) which reduces (2.2) to a first order integrable ordinary differential equation. An exact solution to (2.1) is then obtained by solving this equation. Now, let us recall the Division Theorem:

Division theorem: Suppose that $P(w, z), Q(w, z)$ are polynomials in $C(w, z)$ and $P(w, z)$ is irreducible in $C(w, z)$. If $Q(w, z)$ vanishes at all zero points of $P(w, z)$, then there exists a polynomial $G(w, z)$ in $C(w, z)$ such that

$$Q[w, z] = P[w, z]G[w, z]. \tag{2.6}$$

3. Zoomeron equation

Let us first consider the zoomeron equation (Alquran and Al-Khaled, 2012)

$$\left(\frac{u_{xy}}{u}\right)_{tt} - \left(\frac{u_{xy}}{u}\right)_{xx} + 2(u^2)_{xt} = 0. \tag{3.1}$$

Using the transformation

$$u(x, t) = f(\xi), \qquad \xi = x + cy - wt \tag{3.2}$$

and substituting the Eq. (3.2) into Eq. (3.1) yields

$$cw^2\left(\frac{f''(\xi)}{f(\xi)}\right)'' - c\left(\frac{f''(\xi)}{f(\xi)}\right)'' - 2w(f^2(\xi))'' = 0, \tag{3.3}$$

where the prime denotes the derivation with respect to ξ. Integrating Eq. (3.3) twice and setting first integration constant to zero, we obtain

$$c(w^2 - 1)f''(\xi) - 2wf^3(\xi) - rf(\xi) = 0, \tag{3.4}$$

where r is the integration constant.

Using (2.4) we get

$$\dot{X}(\xi) = Y(\xi) \tag{3.5}$$

$$\dot{Y}(\xi) = \frac{X(\xi)(r + 2wX^2(\xi))}{c(w^2 - 1)}. \tag{3.6}$$

In conformity with the first integral method, we suppose that $X(\xi)$ and $Y(\xi)$ are nontrivial solutions of (3.5) and (3.6), and $q(X, Y) = \sum_{i=0}^{m} a_i(X)Y^i$ is an irreducible polynomial in the complex domain $C[X, Y]$ such that

$$q[X(\xi), Y(\xi)] = \sum_{i=0}^{m} a_i(X)Y^i = 0, \tag{3.7}$$

where $a_i(X), (i = 0, 1, \ldots, m)$ are polynomials of X and $a_m(X) \neq 0$. Eq. (3.7) is named the first integral to (3.5) and (3.6), due to the Division Theorem, there exists a polynomial $g(X) + h(X)Y$ in the complex domain $C[X, Y]$ such that

$$\frac{dq}{d\xi} = \frac{\partial q}{\partial X}\frac{\partial X}{\partial \xi} + \frac{\partial q}{\partial Y}\frac{\partial Y}{\partial \xi} = [g(X) + h(X)Y]\sum_{i=0}^{m} a_i(X)Y^i. \tag{3.8}$$

For this equation, we take two different cases, assuming that $m = 1$ and $m = 2$ in Eq. (3.7).

Case I: Suppose that $m = 1$, by equating the coefficients of $Y^i (i = 0, 1, 2)$ on both sides of Eq. (3.8), we get

$$\dot{a}_1(X) = h(X)a_1(X), \tag{3.9}$$

$$\dot{a}_0(X) = g(X)a_1(X) + h(X)a_0(X), \tag{3.10}$$

$$a_1(X)\dot{Y} = g(X)a_0(X) = a_1(X)\left(\frac{X(r + 2wX^2)}{c(w^2 - 1)}\right). \tag{3.11}$$

Since $a_i(X)$ $(i = 0, 1)$ are polynomials, then from (3.9) we derive that $a_1(X)$ is constant and $h(X) = 0$. So as to make simpler calculations, we take $a_1(X) = 1$. Balancing the degrees of $g(X)$ and $a_0(X)$, we conclude that $deg(g(X)) = 1$ only. Suppose that $g(X) = A_1X + B_0$, and $A_1 \neq 0$, then we find $a_0(X)$

$$a_0(X) = \frac{A_1}{2}X^2 + B_0X + A_0. \tag{3.12}$$

Substituting $a_0(X), a_1(X)$ and $g(X)$ into Eq. (3.11) and setting all the coefficients of powers X to be zero, then we gain a system of nonlinear algebraic equations and by solving it, we obtain

$$A_0 = \frac{r}{2\sqrt{wc(w^2-1)}}, \quad A_1 = 2\sqrt{\frac{w}{c(w^2-1)}}, \quad B_0 = 0. \tag{3.13}$$

Using (3.13) into (3.7), we obtain

$$Y(\xi) = -\frac{r}{2\sqrt{wc(w^2-1)}} - \sqrt{\frac{w}{c(w^2-1)}}X^2(\xi). \tag{3.14}$$

Combining (3.14) with (3.5), we obtain the exact solution to (3.4) and then the exact solution can be written as:

$$X(\xi) = \sqrt{\frac{-r}{2w}}\tanh\left(\frac{\sqrt{r}(\xi + C_1)}{\sqrt{2c(1-w^2)}}\right), \tag{3.15}$$

where C_1 is integration constant. Thus the solitary wave solution to the Zoomeron Eq. (3.1) can be written as:

$$u(x,t) = \sqrt{\frac{-r}{2w}}\tanh\left(\frac{\sqrt{r}(x + cy - wt + C_1)}{\sqrt{2c(1-w^2)}}\right). \tag{3.16}$$

Case II: Suppose that $m = 2$, by equating the coefficients of $Y^i (i = 0, 1, 2, 3)$ on both sides of Eq. (3.8), we get

$$\dot{a}_2(X) = h(X)a_2(X), \tag{3.17}$$

$$\dot{a}_1(X) = g(X)a_2(X) + h(X)a_1(X), \tag{3.18}$$

$$\dot{a}_0(X) = -2a_2(X)\left(\frac{X(r + 2wX^2)}{c(w^2-1)}\right) + g(X)a_1(X) + h(X)a_0(X), \tag{3.19}$$

$$a_1(X)\dot{Y} = g(X)a_0(X) = a_1(X)\left(\frac{X(r + 2wX^2)}{c(w^2-1)}\right). \tag{3.20}$$

Considering the fact that $a_2(X)$ is a polynomial of X, then from (3.17) we derive that $a_2(X)$ is constant and $h(X) = 0$. So as to make simpler calculations, take $a_2(X) = 1$. Balancing the degrees of $g(X)$ and $a_0(X)$, we conclude that $deg(g(X)) = 1$ only. Assume that $g(X) = A_1X + B_0$, and $A_1 \neq 0$, then we find $a_1(X)$ and $a_0(X)$ as

$$a_1(X) = \frac{A_1}{2}X^2 + B_0X + A_0. \tag{3.21}$$

$$a_0(X) = \left(\frac{A_1^2}{8} - \frac{w}{c(w^2-1)}\right)X^4 + \frac{A_1B_0}{2}X^3 + \left(-\frac{r}{c(w^2-1)} + \frac{A_1A_0}{2} + \frac{B_0^2}{2}\right)X^2 + B_0A_0X + d. \tag{3.22}$$

Substituting $a_0(X), a_1(X), a_2(X)$ and $g(X)$ in Eq. (3.20) and setting all the coefficients of powers X to be zero, then we gain a system of nonlinear algebraic equations and by solving it, we obtain

$$A_0 = \frac{r}{w}\sqrt{\frac{w}{c(w^2-1)}}, \quad A_1 = 4\sqrt{\frac{w}{c(w^2-1)}}, \quad B_0 = 0 \quad d = \frac{r^2}{4cw(w^2-1)} \tag{3.23}$$

Using (3.23) into (3.11), we find

$$Y(\xi) = -\frac{1}{2w}\left(\sqrt{\frac{w}{c(w^2-1)}}\right)(r + 2wX^2(\xi)). \tag{3.24}$$

Combining (3.24) with (3.5), we obtain the exact solution to (3.4) and then the exact solution can be written as:

$$X(\xi) = -\sqrt{\frac{r}{2w}}\tan\left(\frac{\sqrt{r}(\xi + C_1)}{\sqrt{2c(w^2-1)}}\right), \tag{3.25}$$

where C_2 is integration constant. Thus the periodic wave solutions to the Zoomeron Eq. (3.1) can be written as:

$$u(x,t) = -\sqrt{\frac{r}{2w}}\tan\left(\frac{\sqrt{r}(x + cy - wt + C_1)}{\sqrt{2c(w^2-1)}}\right). \tag{3.26}$$

As a result, we find periodic wave and soliton solutions of the Zoomeron equation different from the solutions found in (Alquran and Al-Khaled, 2012; Irshad and Mohyud-Din, in press; Abazari, 2011).

4. The Klein–Gordon–Zakharov equations

In the theoretical investigation of the dynamics of strong Langmuir turbulence in plasma physics, various Zakharov equations take an important role (Thornhill and Haar, 1978; Dendy, 1990). We consider the following Klein-Gordon-Zakharov equations:

$$\begin{aligned} &u_{tt} - u_{xx} + u + \alpha vu = 0, \\ &v_{tt} - v_{xx} = \beta(|u|^2)_{xx}, \end{aligned} \tag{4.1}$$

with u is a complex function and v is a real function , where α, β are two nonzero real parameters. This system describes the interaction of the Langmuir wave and the ion acoustic wave in a high frequency plasma.

Using the wave variable

$$u(x,t) = e^{i(kx+wt+\xi_0)}\varphi(x,t), \tag{4.2}$$

where $\varphi(x,t)$ is a real-valued function, k, w are two real constants to be determined, ξ_0 is an arbitrary constant. Then Eq. (4.1) is carried to a PDE system

$$\begin{aligned} &\varphi_{tt} - \varphi_{xx} + (k^2 - w^2 + 1)\varphi + \alpha v\varphi = 0, \\ &w\varphi_t - k\varphi_x = 0, \\ &v_{tt} - v_{xx} = \beta(\varphi^2)_{xx}. \end{aligned} \tag{4.3}$$

We take

$$\varphi(x,t) = \varphi(\xi) = \varphi(wx + kt + \xi_1), \tag{4.4}$$

where ξ_1 is an arbitrary constant. Substituting (4.4) into (4.3), we infer that

$$v(x,t) = \frac{(w^2 - k^2)\varphi''(\xi)}{\alpha\varphi(\xi)} + \frac{(w^2 - k^2 - 1)}{\alpha}. \tag{4.5}$$

Therefore, we can also assume

$$v(x,t) = \psi(wx + kt + \xi_1). \tag{4.6}$$

Substituting (4.6) into last equation in (4.3) and integrating the resultant equation twice with respect to ξ, we derive

$$\psi(\xi) = \frac{\beta w^2\varphi^2(\xi)}{k^2 - w^2} + C, \tag{4.7}$$

where C is an integration constant. Substituting (4.7) into the first equation in (4.3), we get

$$\varphi''(\xi)+\frac{(k^2-w^2+1+\alpha C)}{(k^2-w^2)}\varphi(\xi)+\frac{\alpha\beta w^2}{(k^2-w^2)^2}\varphi^3(\xi)=0. \tag{4.8}$$

Let $l=\frac{(k^2-w^2+1+\alpha C)}{(k^2-w^2)}$, $m=\frac{\alpha\beta w^2}{(k^2-w^2)^2}$, thus (4.8) becomes the Lienard equation

$$\varphi''+l\varphi+m\varphi^3=0. \tag{4.9}$$

In the following we will discuss how to solve exactly the Lienard Eq. (4.9). Using (2.4) we get

$$\dot{X}(\xi)=Y(\xi), \tag{4.10}$$

$$\dot{Y}(\xi)=-lX(\xi)-mX^3(\xi). \tag{4.11}$$

In conformity with the first integral method, we suppose that $X(\xi)$ and $Y(\xi)$ are nontrivial solutions of (4.10) and (4.11), and $q(X,Y)=\sum_{i=0}^{m}a_i(X)Y^i$ is an irreducible polynomial in the complex domain $C[X,Y]$ such that

$$q[X(\xi),Y(\xi)]=\sum_{i=0}^{m}a_i(X)Y^i=0, \tag{4.12}$$

where $a_i(X),(i=0,1,\ldots,m)$ are polynomials of X and $a_m(X)\neq 0$. Eq. (4.12) is named the first integral to (4.10) and (4.11), due to the Division Theorem, there exists a polynomial $g(X)+h(X)Y$ in the complex domain $C[X,Y]$ such that

$$\frac{dq}{d\xi}=\frac{\partial q}{\partial X}\frac{\partial X}{\partial \xi}+\frac{\partial q}{\partial Y}\frac{\partial Y}{\partial \xi},=[g(X)+h(X)Y]\sum_{i=0}^{m}a_i(X)Y^i. \tag{4.13}$$

In this example, we take two different cases, assuming that $m=1$ and $m=2$ in Eq. (4.12).

Case I: Suppose that $m=1$, by equating the coefficients of $Y^i(i=0,1,2)$ on both sides of Eq. (4.13), we have

$$\dot{a}_1(X)=h(X)a_1(X), \tag{4.14}$$

$$\dot{a}_0(X)=g(X)+h(X)a_0(X), \tag{4.15}$$

$$a_1(X)\dot{Y}=g(X)a_0(X)=a_1(X)(-lX-mX^3). \tag{4.16}$$

Considering the fact that $a_i(X)\ (i=0,1)$ are polynomials, then from (4.14) we deduce that $a_1(X)$ is constant and $h(X)=0$. So as to make simpler calculations, we choose $a_1(X)=1$. Balancing the degrees of $g(X)$ and $a_0(X)$, we conclude that $deg(g(X))=1$ only. Take $g(X)=A_1X+B_0$, and $A_1\neq 0$, then we find $a_0(X)$

$$a_0(X)=\frac{A_1}{2}X^2+B_0X+A_0. \tag{4.17}$$

Substituting $a_0(X),a_1(X)$ and $g(X)$ in Eq. (4.16) and setting all the coefficients of powers X to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$A_1=\pm\sqrt{2m}i,\quad B_0=0,\quad A_0=\frac{l}{\sqrt{2m}}i. \tag{4.18}$$

where A_0 is free parameter. Using Eq. (4.18) into Eq. (4.12), we obtain

$$Y(\xi)=\mp\frac{\sqrt{2}}{2\sqrt{m}}i(mX^2(\xi)+l), \tag{4.19}$$

Combining (4.19) with (4.10), we obtain the exact solution to (4.11) and then the exact solution to the Lienard Eq. (4.9) can be written as:

$$X(\xi)=\varphi(\xi)=\pm\sqrt{\frac{-l}{m}}\coth\left(\sqrt{\frac{l}{2}}\xi\right), \tag{4.20}$$

then the soliton solutions to the Klein–Gordon–Zakharov Eq. (4.1) can be written as:

$$\begin{aligned}u(x,t)&=\pm\sqrt{\frac{-l}{m}}e^{i(kx+wt+\xi_0)}\coth\left[\sqrt{\frac{l}{2}}(kx+wt+\xi_1)\right].\\ v(x,t)&=\mp\frac{\beta w^2 l}{m(k^2-w^2)}e^{2i(kx+wt+\xi_0)}\coth^2\left[\sqrt{\frac{l}{2}}(kx+wt+\xi_1)\right]+C\end{aligned} \tag{4.21}$$

Case II: Suppose that $m=2$, by equating the coefficients of $Y^i(i=0,1,2,3)$ on both sides of Eq. (4.13), we find

$$\dot{a}_2(X)=h(X)a_2(X), \tag{4.22}$$

$$\dot{a}_1(X)=g(X)a_2(X)+h(X)a_1(X), \tag{4.23}$$

$$\dot{a}_0(X)=-2a_2(X)\dot{Y}+g(X)a_1(X)+h(X)a_0(X), \tag{4.24}$$

$$a_1(X)\dot{Y}=g(X)a_0(X)=a_1(X)(-lX-mX^3). \tag{4.25}$$

Considering the fact that $a_2(X)$ is a polynomial of X, then from (4.22) we deduce that $a_2(X)$ is constant and $h(X)=0$. For simplicity, take $a_2(X)=1$. Balancing the degrees of $g(X)$ and $a_0(X)$, we conclude that $deg(g(X))=1$ only. Suppose that $g(X)=A_1X+B_0$, and $A_1\neq 0$, then we find $a_1(X)$ and $a_0(X)$ as

$$a_1(X)=\frac{A_1}{2}X^2+B_0X+A_0. \tag{4.26}$$

$$\begin{aligned}a_0(X)=&\left(\frac{A_1^3}{8}+A_1m\right)X^5+\left(\frac{5A_1^2B_0}{8}+\frac{3B_0m}{2}\right)X^4\\&+\left(\frac{3lA_1}{2}+A_0m+A_1B_0^2+\frac{A_1^2A_0}{2}\right)X^3\\&+\left(\frac{3A_0A_1B_0}{2}+\frac{B_0^3}{2}+2lB_0\right)X^2\\&+\left(A_0B_0^2+lA_0+dA_1\right)X+B_0d.\end{aligned} \tag{4.27}$$

Substituting $a_0(X),a_1(X),a_2(X)$ and $g(X)$ in Eq. (4.25) and setting all the coefficients of powers X to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$A_0=\pm\frac{\sqrt{2}l}{\sqrt{m}}i,\quad A_1=\pm 2\sqrt{2m}i,\quad B_0=0,\quad d=-\frac{l^2}{2m}. \tag{4.28}$$

where A_0 is free parameter. Using Eq. (4.28) into Eq. (4.12), we obtain

$$Y(\xi)=\pm\frac{\sqrt{2}i}{2\sqrt{m}}[mX^2(\xi)+l]. \tag{4.29}$$

Combining (4.29) with (4.10), we obtain the exact solution to (4.11) and then the exact solution to the Lienard Eq. (4.9) can be written as:

$$X(\xi)=\varphi(\xi)=\pm\sqrt{-\frac{l}{m}}\tanh\left(\sqrt{\frac{l}{2}}\xi\right), \tag{4.30}$$

then the traveling wave solution to the Klein–Gordon–Zakharov equation (4.1) can be written as:

$$u(x,t) = \mp\sqrt{\frac{-l}{m}}e^{i(kx+wt+\xi_0)}\tanh\left[\sqrt{\frac{l}{2}}(kx+wt+\xi_1)\right].$$
$$v(x,t) = \pm\frac{\beta w^2 l}{m(k^2-w^2)}e^{2i(kx+wt+\xi_0)}\tanh^2\left[\sqrt{\frac{l}{2}}(kx+wt+\xi_1)\right] + C. \tag{4.31}$$

Our results (4.21) and (4.31) can be compared with the result of Ebadi et al. (2010, 2012), Shang et al. (2008), Ismail and Biswas (2010) and Song et al. (2013a,b).

5. Conclusion

The first integral method has been successfully utilized to establish new soliton solutions. The applicability of this method is reliable and effective and gives more solutions. Thus, we deduce that the referred method can be extended to solve many systems of nonlinear partial differential equations which are arising in the theory of solitons and other areas such as physics, biology, and chemistry. With the help of symbolic computation (Maple), a rich variety of exact solutions are obtained by applying first integral method, and the method can be applied to other nonlinear evolution equations.

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