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TECHNICAL PAPER

Prediction of stable depth of cuts in turning and milling operations: a new probabilistic approach

M. Alper Sofuoğlu[1]

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Abstract

Chatter vibration in turning and milling operations is one of the most critical problems that causes low workpiece quality and manufacturing efficiency. Therefore, the determination of stable cutting depths is crucial for these operations. Several vibrational characteristics [natural frequency (ω_n), stiffness coefficient (k), and damping coefficient (s)] affect stable cutting depths. The vibration characteristics of these operations show randomness for every setup condition. For this reason, the randomness of the vibration characteristics should be modeled. In this study, a probabilistic approach and regression model are combined for turning operation. Also, a probabilistic approach and analytical model are integrated for milling operation. The purpose of these models is to establish confidence intervals for stability diagrams. As a result, the operators can work in a secure region during the operations.

Keywords Chatter vibrations · Stable cutting depths · Stochastic approach · Turning operation · Milling operation

List of symbols

a_{lim}	Axial depth of cut (chatter-free)
CNC	Computer numerical control
D	Test statistic for Kolmogorov–Smirnov test
E_i	Expected frequency of ith data
F	Theoretical cumulative distribution of the distribution
FOSM	First-order second-moment method
k	Stiffness coefficient
k_x	Stiffness coefficient in the x-direction
k_y	Stiffness coefficient in the y-direction
K	Imaginary part of eigenvalue/real part of the eigenvalue
K_t	Radial cutting constant
N	The number of data
N_t	The number of teeth
O_i	Observed frequency of ith data
s	Damping coefficient
s_x	Damping coefficient in the x-direction
s_y	Damping coefficient in the y-direction
S	Test statistic for Anderson–Darling test
ω_n	Natural frequency
ω_{nx}	Natural frequency in the x-direction
ω_{ny}	Natural frequency in the y-direction
χ^2	Test statistic for Chi-square test
α	Significance level
$\lambda_{r,i}$	Eigenvalue (real and imaginary parts)

1 Introduction

The calculation of the stability boundary of different machining operations requires modeling and parameterization of cutting forces/dynamic machine characteristics. The uncertainty of machining parameters causes shifts in the stability lobe diagram. Several researchers have proposed different methods to calculate confidence levels of stability lobe diagrams. Kong et al. [1] used a fuzzy method, and Totis [2] studied a probabilistic approach to describe parameter uncertainty. Duncan et al. [3] investigated the effect of random parameters on the milling stability lobe diagrams. Graham et al. [4] established a robust chatter stability model. The ambiguity in the natural frequency and the cutting coefficient was considered in the model. Sims et al. [5] used fuzzy arithmetic methods for the chatter stability problem. To obtain solutions for process design problems, the fuzzy arithmetic is used. Zhang [6] found a speed optimization formula for

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✉ M. Alper Sofuoğlu
asofuoglu@ogu.edu.tr

[1] Mechanical Engineering Department, Eskişehir Osmangazi University, Eskisehir, Turkey

the ambiguous factors in a milling operation. First-order second-moment method (FOSM) is one of the most popular computational methods used by researchers [7]. Recently, several authors have investigated some drawbacks of FOSM. The fourth-moment method was offered to improve FOSM [8, 9]. Reliability analysis of static and dynamic structural systems has been developed in recent years. The reliability of dynamic structural systems generally contains structural response overrun and fatigue created [10–14].

Several methods of predicting chatter stability lobes assumed that vibrational parameters are constant. Nonetheless, in practical applications, these parameters can change [15–24]. Thomas et al. [15] studied the effects of cutting parameters on the cutting force, tool vibration, and change in modal parameters. Lee et al. [16] studied the change in tool point dynamics describing the dynamics of the tool–holder–spindle system and machining stability. Liu et al. [17] investigated the probability distributions of modal parameters with an experimental study. Sahali et al. [18] proposed a genetic algorithm to perform a multi-response robust optimization. In the machining operation, interval/fuzzy theory was used to show the uncertainty. Several studies were also conducted for reliability analysis [19–24]. Shin and Lee [19] conducted a reliability analysis of roadway minimum radius design. TruckSim software was used to develop an analytical model. The first-order reliability technique and numerical researches were used to calculate accident probability. Li et al. [20] proposed an indexing accuracy reliability and its sensitivity of power tool turret. Li et al. [21] offered a reliability optimization to increase reliability robustness and improve the stochastic static deformation of computer numerical control (CNC) lathe. Chai et al. [22] performed a reliability analysis of a vessel in random beam seas. Also, the stochastic roll response was studied, and the 4D path integration method was used. Löser and Großmann [23] proposed a time-efficient technique to obtain confidence interval levels of stability lobe diagrams for the milling process. An experimental study was carried out to compare the results. The purpose of the technique is to decrease computational time. Huang et al. [24] investigated the chatter stability in turning using probabilistic analysis. Monte Carlo simulation and advanced first-order second-moment method were used to predict the reliability of stability. Mass, stiffness, and cutting coefficients were considered during the analysis. After conducting a literature survey, this study hybridizes a probabilistic approach with analytical and regression models to determine confidence intervals of stability diagrams for the first time in the literature.

In this study, a probabilistic approach was used to predict stable cutting depths. Turning and milling operations were investigated. A probabilistic approach was modeled by using vibrational characteristics [natural frequency (ω_n), stiffness coefficient (k), and damping coefficient (s)] affecting stable cutting depths in turning and milling operations. A previous experimental study was used to develop a probabilistic model [25]. Türkeş [25] proposed a process damping model to predict stability in turning and milling operations. Several experiments were conducted in his study. Also, an experimental study was carried out for the turning operations. Using these experimental data, probabilistic approach and regression model were combined for turning operation. Also, the probabilistic approach and analytical model were integrated for the milling operation to establish confidence intervals of stability diagrams. It has been observed that in the studies performed in the literature, some assumptions about the distribution of vibration characteristics have been studied, but no detailed analysis of these characteristics has been performed. To the best of our knowledge, Mersenne Twister algorithm was used in random number generation for vibration characteristics, and no study has been found. The methods (goodness-of-fit tests) are given in Sect. 2. Section 3 provides details of the mathematical aspects of the study. The results are given in Sect. 4. Section 5 includes the conclusions.

2 Methods

2.1 The goodness-of-fit tests

In order to determine the appropriate distribution, the methods are explained below.

2.1.1 Kolmogorov–Smirnov test

The Kolmogorov–Smirnov test is based on the empirical distribution function. Given n ordered data points (Y_1, Y_2,…, Y_n), the empirical distribution function is defined as $E_n = n(i)/N$, where $n(i)$ is the number of points less than Y_i and Y_i are ordered from smallest to largest value. This is a step function that increases by $1/N$ at the value of each ordered data point [26].

The test statistic is calculated using Eq. 1:

H_0: The data have a specific distribution.

H_1: The data do not have a specific distribution.

$$D = \max \left\| F\left(Y_i\right) - \frac{i}{N} \right\| \quad 1 \le i \le N \tag{1}$$

where F is the theoretical cumulative distribution of the distribution The hypothesis is rejected if the test statistic is higher than the critical value.

2.1.2 Anderson–Darling test

Anderson–Darling test is an adjustment of the Kolmogorov–Smirnov test and gives more weight to the tails than does the Kolmogorov–Smirnov test. The Kolmogorov–Smirnov test is distribution-free in the sense that the critical values do not

depend on the tested specific distribution. The Anderson–Darling test makes use of the particular distribution in computing critical values (Eq. 2). The Anderson–Darling test statistic is defined by $A^2 = -N-S$:

$$S = \sum_{i=1}^{N} \frac{2i-1}{N} \left[\ln F\left(Y_i\right) + \ln\left(1 - F\left(Y_{N+1-i}\right)\right)\right] \tag{2}$$

where F is the cumulative distribution function of the specified distribution, and Y_i are the ordered data. The test is a one-sided test, and the hypothesis that the distribution is of a specific form is rejected if the test statistic is higher than the critical value [26].

2.1.3 Chi-square test

The Chi-square goodness-of-fit test is applied to binned data. The test statistic is calculated in Eq. 3:

$$\chi^2 = \sum_{i=1}^{r} \frac{(O_i - E_i)^2}{E_i} \tag{3}$$

where O_i is the ith observed frequency for the bin and E_i is the ith expected frequency for the bin. The test statistic follows, approximately, a Chi-square distribution with (k–c) degrees of freedom. The hypothesis that the data are from a population with the specified distribution is rejected if $\chi^2 > \chi^2 a,k-c$ where $\chi^2 a,k-c$ the Chi-square percent is point function with (k–c) degrees of freedom and a significance level of the α [26].

3 Experimental/statistical study

To investigate the vibration parameters affecting the stable cutting depth, several data were used. Hammer test results were used to obtain vibrational parameters for both turning and milling operations (Fig. 1). The vibrational parameters of the turning operation are ω_n, k, and s. The vibrational parameters of the milling operation are ω_{nx}, ω_{ny}, s_x, s_y, k_x, and k_y. EasyFit software was used during analysis. The software tests 61 different continuous distributions. Using EasyFit software, the goodness-of-fit test was performed. Kolmogorov–Smirnov, Anderson–Darling, and Chi-square tests were used in this study. The maximum likelihood method was used to determine the parameters of the distributions. Mersenne Twister algorithm is used to derive random numbers for distributions. The algorithm generates pseudorandom numbers at high quality. Pseudorandom number generators have been widely used in several applications, particularly simulation and cryptography. A detailed mathematical presentation is given in references [27, 28]. The algorithm is preferred in many statistical simulations because it provides positive results at high speeds. In this study, 10,000 data are derived.

3.1 Turning operation

The study conducted by the Türkeş [25] was used, and an experimental study was conducted for the turning operation. As a result, hammer test results and stability results of turning operations were used. A probabilistic approach was developed by using hammer test data. Regression analysis of the turning operation was performed by using stability results data. As a result, the probabilistic approach and regression analysis were combined. These statistics of the stiffness coefficient, damping coefficient, and natural frequency values are given in Tables 1, 2, and 3, respectively. The experiments were carried out for AISI-1050 and Al-2024 materials. The results of these experimental tests are shown in Tables 4 and 5.

3.2 Milling operation

The properties and parameters of the cutting tool used for milling operation are given in Table 6. An analytical model was developed for the milling operation. Mathematical details of the analytical model were given in the literature studies [29, 30]. The general formula of chatter-free axial depth of cut is given in Eq. 4. A two-degree-of-freedom milling system was taken into account. In this study, Cut Pro software was used. Also, the hammer test results of Türkeş's study [25] milling operation were used to develop a probabilistic approach. In the final stage, the probabilistic approach and the analytical model were combined.

$$a_{\lim} = -\frac{2\pi\lambda_r}{N_t K_t}\left(1 + K^2\right) \tag{4}$$

where λ_r eigenvalue (imaginary part), K imaginary part of eigenvalue $(\lambda_i)/\lambda_r$, K_t radial cutting constant, and N_t the number of teeth

Table 1 Descriptive statistics of k values (turning)

Mean	6.527e6
Skewness	1.479
Excess kurtosis	0.74607

Table 2 Descriptive statistics of s values (turning)

Mean	0.0386
Skewness	1.2777
Excess kurtosis	1.3663

Table 3 Descriptive statistics w_n values (turning)

Mean	1091.3
Skewness	0.56004
Excess kurtosis	−0.68131

Table 4 Experimental results of stable cutting depth in turning operation (AISI-1050) [25]

The number of revolutions (rpm)	Tool overhang length (mm)	ω_n (Hz)	k (N/m)	s (%)	Stable cutting depth (mm)
90	80	1101.0	1.02E+07	3.97E−02	4.5
125	80	1101.0	1.02E+07	3.97E−02	4.0
180	80	1101.0	1.02E+07	3.97E−02	3.5
250	80	1101.0	1.02E+07	3.97E−02	2.8
355	80	1101.0	1.02E+07	3.97E−02	2.3
500	80	1101.0	1.02E+07	3.97E−02	1.8
710	80	1101.0	1.02E+07	3.97E−02	1.3
90	90	944.6	5.71E+06	4.15E−02	4.0
125	90	944.6	5.71E+06	4.15E−02	3.5
180	90	944.6	5.71E+06	4.15E−02	3.0
250	90	944.6	5.71E+06	4.15E−02	2.4
355	90	944.6	5.71E+06	4.15E−02	2.0
500	90	944.6	5.71E+06	4.15E−02	1.5
710	90	944.6	5.71E+06	4.15E−02	1.1
90	100	839.3	5.24E+06	1.75E−02	3.8
125	100	839.3	5.24E+06	1.75E−02	3.3
180	100	839.3	5.24E+06	1.75E−02	2.8
250	100	839.3	5.24E+06	1.75E−02	2.3
355	100	839.3	5.24E+06	1.75E−02	1.8
500	100	839.3	5.24E+06	1.75E−02	1.3
710	100	839.3	5.24E+06	1.75E−02	1.0

Table 5 Experimental results of stable cutting depth in turning operation (Al-2024 T3)

The number of revolutions (rpm)	Tool overhang length (mm)	ω_n (Hz)	k (N/m)	s (%)	Stable cutting depths (mm)
355	80	1343.2	3.7E+06	6.5E−02	4.7
500	80	1343.2	3.7E+06	6.5E−02	4.1
710	80	1343.2	3.7E+06	6.5E−02	3.5
355	90	1190.1	2.0E+06	7.79E−02	4.3
500	90	1190.1	2.0E+06	7.79E−02	3.6
710	90	1190.1	2.0E+06	7.79E−02	3.1
355	100	971.9	1.4E+06	2.2E−02	3.6
500	100	971.9	1.4E+06	2.2E−02	3.1
710	100	971.9	1.4E+06	2.2E−02	2.6
355	110	849.8	1.1E+06	4.4E−02	3.4
500	110	849.8	1.1E+06	4.4E−02	2.9
710	110	849.8	1.1E+06	4.4E−02	2.4

4 Results and discussion

4.1 The results of turning operation

4.1.1 Probabilistic analysis of vibrational parameters in turning operation

The ranks for the goodness-of-fit test for the coefficient of rigidity (best five distributions) are given in Table 7. The data from Tables 4 and 5 were used. When the sequences are examined, it is seen that the best distribution is the log-logistic (3P) distribution.

The goodness-of-fit test results for the log-logistic distribution (3P) and the parameter values of the distribution according to the maximum likelihood estimation method are given in Table 8. With respect to three different tests, log-logistic (3P) distribution is appropriate.

The ranks for the goodness-of-fit test for the coefficient of damping (best five distributions) are given in Table 9. The data from Tables 4 and 5 were used

Table 6 Cutting tool properties and cutting parameters (milling operation)

Simulation type	Single milling simulation with clockwise spindle direction (face milling)
Cutter type	Cylindrical end
Number of flutes	4 (uniform pitch and uniform helix)
Cutter material with geometry	High-speed steel (HSS) cutter material with radius = 9.525, length = 32 mm, first bearing position = 200 mm, helix = 30°, relief = 5°, rake = 5°
Workpiece material	Aluminum 7075-T6
Feed rate	0.05 mm/flute
Spindle speed	5000 rpm
Axial depth of cut	5 mm
Surface speed (m/min)	299.237
Immersion angles	Entrance = 0°, exit = 50.956°
Sampling frequency scale	10
Divisions along chip thickness	10
Angular divisions on tool	10
Angular increments	8
Tolerance	6%

Table 7 Goodness-of-fit test for k (rankings)

	Kolmogorov–Smirnov	Anderson–Darling	χ^2
1. Log logistic (3P)	1	1	3
2. Log normal (3P)	2	2	1
3. Inverse Gaussian (3P)	4	3	2
4. Fatigue life (3P)	5	8	5
5. Log logistic	3	12	7

Table 8 Results of the goodness-of-fit test for log-logistic distribution and parameter values of the distribution

	Kolmogorov–Smirnov	Anderson–Darling	χ^2
Statistic	0.09251	0.47089	0.51188
p value	0.8225	*	0.99168

Table 9 Goodness-of-fit test for s (rankings)

	Kolmogorov–Smirnov	Anderson–Darling	χ^2
1. Log Pearson (3P)	2	1	11
2. Pearson 5 (3P)	9	7	2
3. Pearson 6 (4P)	10	5	4
4. Pearson 6	3	6	12
5. Frechet (3P)	8	11	3

Table 10 Results of the goodness-of-fit test for log-Pearson distribution and parameter values of the distribution

	Kolmogorov–Smirnov	Anderson–Darling	χ^2
Statistic	0.07447	0.23215	0.97754
p value	0.95656	*	0.91318
Parameter values: $\alpha = 81.437$; $\beta = 0.05505$; $\gamma = -7.8611$			

during the analysis. When the rankings are examined, it is obtained that the best distribution is the log-Pearson (3P) distribution.

The goodness-of-fit test results for the log-Pearson distribution (3P) and the parameter values of the distribution according to the maximum likelihood estimation method are given in Table 10. According to three different tests, log-Pearson (3P) distribution is suitable.

The ranks for the goodness-of-fit test for natural frequency (best five distributions) are given in Table 11. The data from Tables 4 and 5 were used during the calculations. When the sequences are examined, the best distribution is the gamma (3P) distribution.

The goodness-of-fit test results for the gamma distribution (3P) and the parameter values of the distribution according to the maximum likelihood estimation method are given in Table 12. When the p values are taken into account, gamma (3P) distribution is suitable.

4.1.2 Regression analysis results of the turning operation

The regression equations were derived by using the experimental data from Tables 4 and 5. The equations are given below (Eqs. 5–6). Equation 5 is related to Türkeş's study [25] (Table 4), and the other equation is associated with

Table 11 Goodness-of-fit test for w_n (rankings)

	Kolmogorov–Smirnov	Anderson–Darling	χ^2
Gamma 3P	7	1	5
Fatigue 3P	9	2	3
Inverse Gaussian	10	3	4
Pearson 5	14	6	1
Gen. ext. val.	8	4	11

Table 12 Goodness-of-fit test for gamma distribution and parameter values of the distribution

	Kolmogorov–Smirnov	Anderson–Darling	χ^2
Statistic	0.0894	0.38226	0.35365
p value	0.85171	*	0.99651
Parameter values: $\alpha = 4.1233$; $\beta = 155.46$ $\gamma = 450.33$			

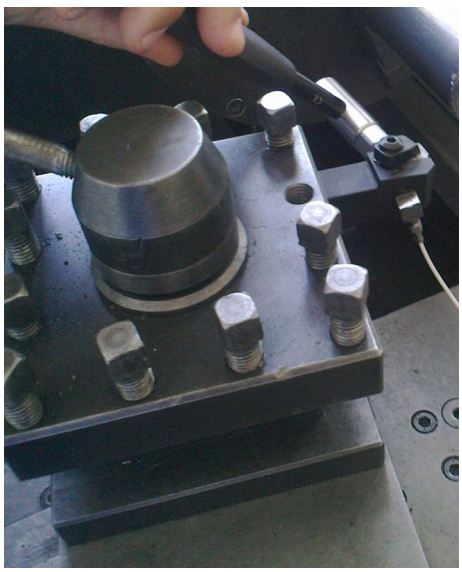


Fig. 1 Hammer test

experimental research. In these equations, stable cutting depth is the dependent variable and number of revolutions, ω_n, k, and s are dependent variables. These equations were derived to obtain confidence interval curves and to show these curves clearly at the same time (Figs. 2, 3).

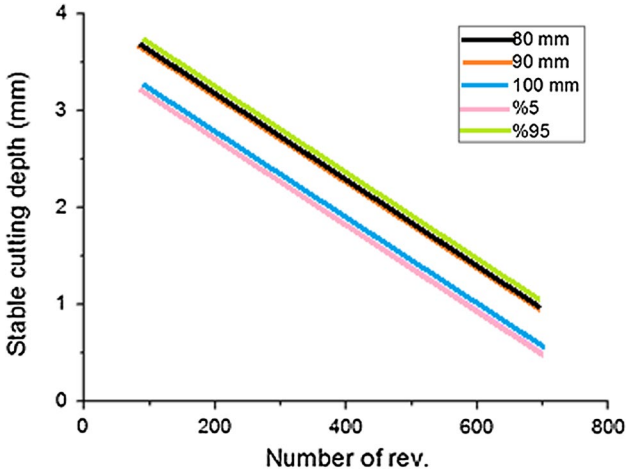


Fig. 2 Stable cutting depths of AISI-1050 material with different confidence levels and experiments

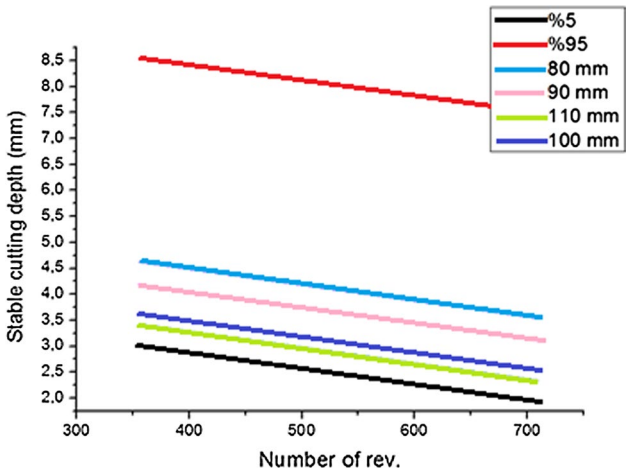


Fig. 3 Stable cutting depths of Al-2024 material with different confidence levels and experiments

$$\begin{aligned}&\text{Stable cutting depth}_1\\&= 0.138\\&\quad -(4.43e-3)\text{number of revolutions}\\&\quad +(5.5e-3)\omega_n\\&\quad -(1.9e-7k-5.06s)\\&\left(R^2=0.89,\ \text{Adj.}\,R^2=0.876,\ S=0.38\right)\end{aligned}\tag{5}$$

$$\begin{aligned}&\text{Stable cutting depth}_2\\&\quad = 2.85\\&\quad -(3.06e-3)\text{number of revolutions}\\&\quad +(1.69e-3)\omega_n\\&\quad -(1.2e-7)k+2.14s\\&R^2=0.987,\ \text{Adj.}\,R^2=0.98,\ S=0.097)\end{aligned}\tag{6}$$

The models fit data well in terms of R^2 values. R^2 values are higher than 89%, and adjusted R^2 values are higher than 87% for the equations, which show that the regression models produce successful results. The confidence level is designated prior to examining the data. Most commonly, the 95% confidence level is used in several areas in the literature. However, the other confidence levels can be used, for example, 90% and 99% [31]. In this study, 5–95% confidence levels are chosen. In Fig. 2, experiments performed in three different tool overhang lengths and graphs obtained at 5–95% confidence levels are shown. It is observed that the experiments took place within the 5–95% confidence levels. Stable cutting depths vary between 0.5 and 3.8 mm.

In Fig. 3, the experiments performed in four different tool overhang lengths and the graphs obtained at the 5–95% confidence level are shown. It is seen that the tests took place within the 5–95% confidence interval. Stable cutting depths vary between 2 and 8.5 mm. As can be seen from Figs. 2 and 3, the confidence interval can be narrow (Fig. 2) and wide (Fig. 3) because different developed distributions affect lower and upper ranges in a different way. The lower number of revolutions and the lower tool overhang length help to reduce chatter vibrations, and they lead to higher stable cutting depths. The slope of the stability curve also affects chatter behavior.

Table 13 Parameters used in milling and the best-fit distributions

Parameters	The best-fit distributions
ω_{nx}	Log Pearson
ω_{ny}	Gen. ext. value
k_x	Error
k_y	Log Pearson
s_x	Johnson SB
s_y	Gen. ext. value

Table 14 Parameters of specified distributions

Distributions	Parameter values
Log Pearson	$\alpha=3.6799\ \beta=-0.01807\ \gamma=6.685$
Gen. ext. value	k = −0.17713 $\sigma=484.47\ \mu=2546.9$
Error	$k=100$ $\sigma=9.047\text{e}6\ \mu=2.345\text{e}7$
Log Pearson	$\alpha=1.2227\ \beta=0.49738\ \gamma=17.157$
Johnson SB	$\gamma=-1.5623\quad \delta=4.1533\quad \lambda=0.27201$ $\zeta=-0.06202$
Gen. ext. value	$k=-1.0736$ $\sigma=0.00902\ \mu=0.03344$

4.2 The results of the milling operation

4.2.1 Probabilistic analysis of vibrational parameters in the milling operation

The most appropriate distributions according to the three different tests for the milling parameters are given in Table 13 using the data in Table 6.

The parameters of the distributions according to the maximum likelihood estimation method are given in Table 14.

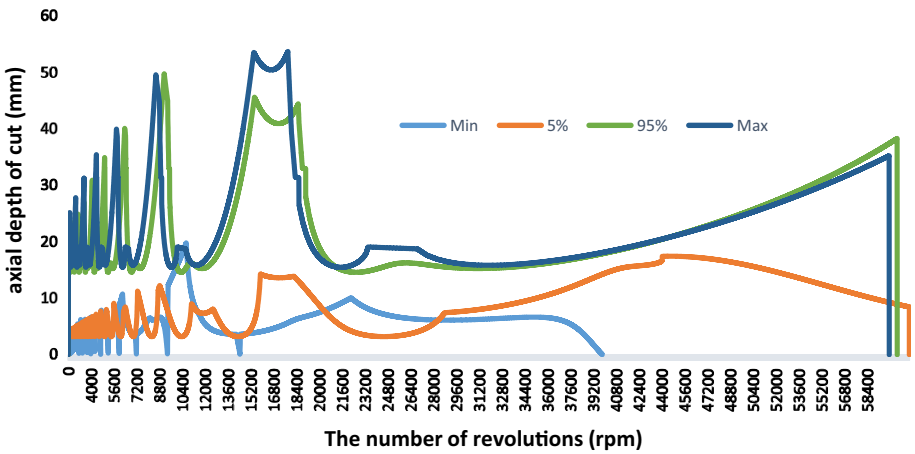


Fig. 4 Stability lobes at minimum, maximum, and different confidence levels (milling)

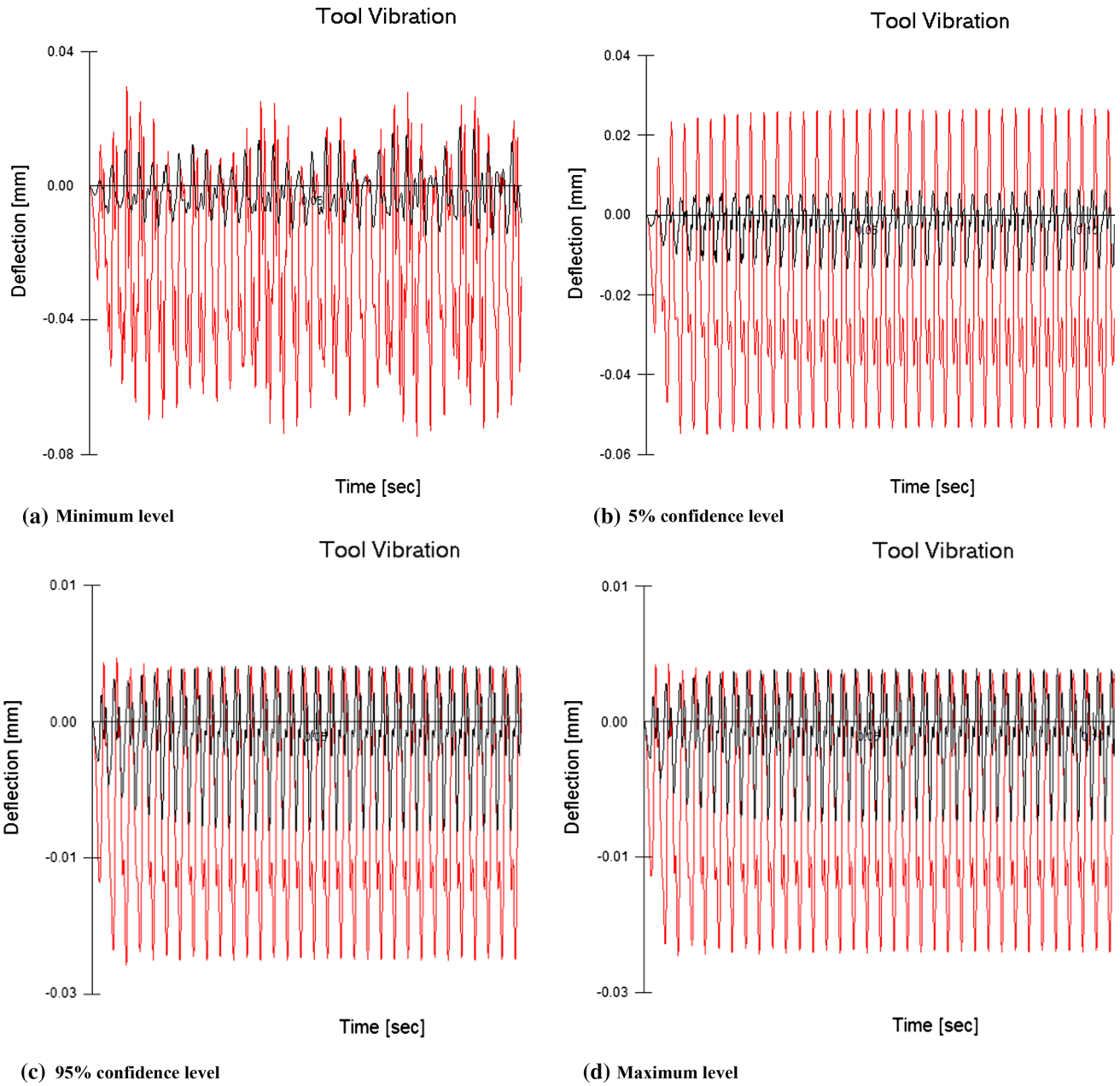


Fig. 5 Time-dependent tool vibration charts (milling)

4.2.2 Analytical model results of the milling operation

Figure 4 shows stability graphs at different confidence levels and minimum–maximum intervals. When the figure is examined, it is seen that it reaches the maximum limit (nearly 50 mm in the range of 15,000–19,000 rpm). This means that milling operation can be carried out without chatter vibrations until the cutting depth reaches 50 mm. When it exceeds, chatter vibration occurs, and it affects surface quality negatively. In the same cycle range, the minimum stable cutting depth varies between 5 and 10 mm. The results show that the vibrational parameters (k, s, ω_n) and the rigidity of the tool setups affect stable cutting depths significantly. It is calculated that the maximum stable cutting depth difference is obtained in the range of 15,000–19,000 rpm. It is a feasible cutting area as can be seen from the graph. Also, 7000–8000 rpm is another available area as can be seen from Fig. 4. After 40,000 rpm, chatter vibration can be observed at the beginning of the operation in terms of minimum stable cutting depths graph. As a result, operators

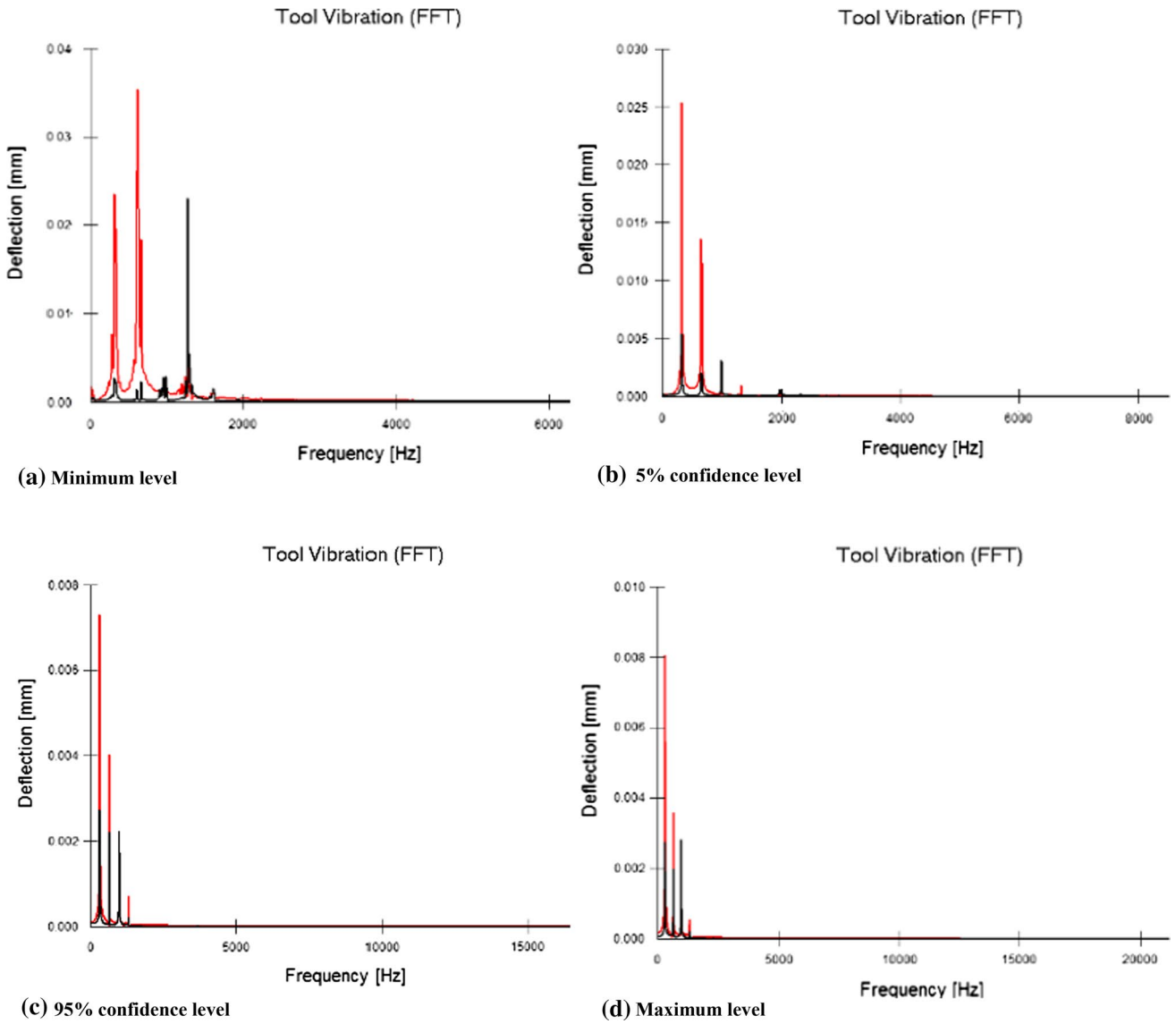


Fig. 6 FFT diagrams of tool vibration charts (milling)

and engineers should measure the vibrational parameters and consider them to prevent chatter vibrations during milling operations. This graph helps to find a secure region without chatter, and randomness of the vibrational parameters is taken into consideration.

In Figs. 5 and 6, time-dependent vibration plots and fast Fourier transform (FFT) graphs are given at minimum–maximum intervals and different confidence levels. The deflection values are 0.06 at the minimum value, while they are 0.02 at the maximum value. To convert time domain to frequency domain, FFT graphs were created. It is seen that the peaks obtained in the FFT graphs are about 300–350 Hz. Obtained maximum deflection values decreased from 0.035 to 0.008. Frequency range shifts to the left, and deflection values decrease from minimum to maximum level. During the machining, this frequency range is detrimental and should be avoided.

5 Conclusions

In this study, a probabilistic approach was proposed to predict stable cutting depths in turning and milling operations. Regression analysis was performed for the turning process using experimental studies. Also, an analytical model was developed for the milling operation. Confidence levels were determined by both turning and milling operations. This study helps engineers and operators to prevent chatter vibrations using confidence level stability diagrams in the machining environment. In future studies, the approach of

confidence level might be used for the other milling (end milling, up milling, and down milling) and drilling operations. Different integrated models (probabilistic–analytical) can be developed. Also, different probabilistic–analytical (1D) models can be developed for turning operation.

References

1. Kong F, Yu J (2007) Study of fuzzy stochastic limited cutting width on chatter. Int J Adv Manuf Technol 33:677–683
2. Tottis G (2009) RCPM-a new method for robust chatter prediction in milling. Int J Mach Tools Manuf 49:73–284
3. Duncan GS, Kurdi M, Schmitz Synder J (2006) Uncertainty propagation for selected analytical milling stability limit analyses. Trans NAMRI/SME 34:17–24
4. Graham E, Mehrpouya M, Park SS (2013) Robust prediction of chatter stability in milling based on the analytical chatter stability. J Manuf Process 15:508–517
5. Sims ND, Manson G, Mann B (2010) Fuzzy stability analysis of regenerative chatter in milling. J Sound Vib 329:1025–1041
6. Zhang X, Zhu L, Zhang D, Ding H, Xiong Y (2012) Numerical robust optimization of spindle speed for milling process with uncertainties. Int J Mach Tools Manuf 61:9–19
7. Zhang M (2009) Structural reliability analysis-method and procedures. Science Press, Beijing, pp 19–61
8. Xu L, Cheng G (2003) Discussion on : moment methods for structural reliability. Struct Saf 25:193–199
9. Zhao YG, Ono T (2001) Moment methods for structural reliability. Struct Saf 23:47–75
10. Bergman LA, Heinrich JC (1982) On the reliability of the linear oscillator and systems of coupled oscillators. Int J Numer Methods Eng 18:1271–1295
11. Spencer BF, Elishakoff I (1988) Reliability of uncertain linear and nonlinear systems. J Eng Mech 114:135–148
12. Haugen EB (1980) Probabilistic mechanical design. Wiley, New York
13. Zhang YM, Wen BC, Liu QL (1998) First passage of uncertain single degree-of-freedom nonlinear oscillators. Comput Methods Appl Mech Eng 165(1998):223–231
14. Zhang YM, Lü CM, Zhou N, Su CQ (2010) Frequency reliability sensitivity for dynamic structural systems. Mech Based Des Struct Mach 38:74–85
15. Thomas M, Beauchamp Y (2003) Statistical investigation of modal parameters of cutting tools in dry turning. Int J Mach Tools Manuf 43(11):1093–1106
16. Lee KJ, Donmez MA (2007) Repeatability analysis on the tool point dynamics for investigation on uncertainty in milling stability. ASME 2007 International Mechanical Engineering Congress and Exposition, 11–15 November, Washington, USA
17. Liu Y, Li TX, Liu K, Zhang YM (2016) Chatter reliability prediction of turning process system with uncertainties. Mech Syst Signal Proc 66(1):232–247
18. Sahali MA, Belaidi I, Serra R (2015) Efficient genetic algorithm for multi-objective robust optimization of machining parameters with taking into account uncertainties. Int J Adv Manuf Technol 77(1–4):677–688
19. Shin J, Lee I (2015) Reliability analysis and reliability-based design optimization of roadway horizontal curves using a first-order reliability method. Eng Optimiz 47(5):622–641
20. Li CY, Wang W, Zhang YM, Guo S, Li ZY, Qiao CS (2015) Indexing accuracy reliability sensitivity analysis of power tool turret. Eksploat Niezawodn 17(1):27–34
21. Li CY, Guo S, Zhang YM, Shi YL (2016) Reliability optimization design for stochastic static deformation of computer numerical control lathe spindle. Proc Inst Mech Eng Part B J Eng Manuf 230(1):83–90
22. Chai W, Naess A, Leira BJ (2015) Stochastic dynamic analysis and reliability of a vessel rolling in random beam seas. J Ship Res 59(2):113–131
23. Löser M, Großmann K (2016) Influence of parameter uncertainties on the computation of stability lobe diagrams. Proc CIRP 46:460–463
24. Huang X, Hu M, Zhang Y, Lv C (2016) Probabilistic analysis of chatter stability in turning. Int J Adv Manuf Technol 87(12):3225–3232
25. Türkeş E (2007) Theoretical and experimental analysis of process damping in machine tool chatter vibration. PhD. Thesis, University of Osmangazi, Eskisehir, Turkey
26. Yılmaz V, Çelik HE (2008) Statistical approach to estimate the wind speed distribution: the case of gelibolu region. Doğuş Univ J 9(1):122–132
27. Mersenne Twister: A Very Fast Random Number Generator. http://www.math.sci.hiroshima-u.ac.jp/~m-mat/MT/emt.html. Accessed 1 Aug 2018
28. The Mersenne Twister. http://www.quadibloc.com/crypto/co4814.html. Accessed 1 Aug 2018
29. Altıntaş Y, Budak E (1995) Analytical prediction of stability lobes in milling. Ann CIRP 44(1):357–362
30. Altıntaş Y (2012) Manufacturing automation: metal cutting mechanics, machine tool vibrations and CNC design, 2nd edn. Cambridge University Press, Cambridge
31. Zar JH (1999) Biostatistical analysis, 4th edn. Prentice Hall, Upper Saddle River, pp 43–45

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