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An Optimal Power Dispatch Technique Using Spot Price of Electricity for Multi-Area Interconnected Systems

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Abstract

In this study, an optimal power dispatch technique using spot price of electricity for multi-area interconnected systems is proposed. It is assumed that all areas in multi-area system agreed upon the formulation of a base spot price for electricity in advance rather than agreeing upon a transfer schedule at a fixed price. This results in a flexible transfer schedule among them. The proposed base spot price of electricity is a linear function of frequency error and integral of frequency error of a system. As long as the tie-line losses are ignored in the system, using the base spot price in the algorithm is enough to find the optimum generation of each areas.

Later, the dispatch algorithm is extended to lossy multi-area case and demonstrated on a five area interconnected system. Inclusion of the tie-line transmission losses is achived by modification of the base spot pirce of electricity. A transfer power algorithm is developed to handle optimal power transfers for the given area transferrable powers among the areas. The dispatch algorithm can handle different system configurations where some tie-lines in the system can be taken out, generation and tie-line transmission constraints. The proposed technique gives the most economical generation of each area together with the most economical energy exchange among them. It results in frequency regulation as well as economic dispatch.

1. Introduction

Interconnection and pooling among power utilities were established primarily because of the advantages in reliability consideration [1,2]. After energy shortages in the 1970s, interest in the economic benefits of power pooling and multi-area interconnection has become very strong, so research on this subject has been very attractive.

Multi-area dispatch problem was solved by considering different aspects of the problem such as tie-line loading restrictions, unit forced outages [3,4] and power pooling [5,7]. Different solution techniques like high-low algorithm [6], production cost simulation [7] and others [8,9] were used in the solution of the problem.

In the literature, electric energy has been treated as a commodity which can be bought and sold in a energy market place. This trend resulted in using pricing of electricity having different formulation in

the solution of different multi-area problems. In reference [10], it is used to solve system security problem. Comparison of different pricing schemes under uncertainty is given in reference [11]. Pricing for transmission in open transmission access problem is discussed in reference [12]. Detailed explanation about spot pricing of electricity can be found in reference [13].

F.C. Schweppe and A.W. Berger studied the effect of real time pricing (they used real time pricing instead of spot pricing) to assist in the control of frequency and tie-line deviations in two area system [14]. To maximize the social welfare under the constraints which are linear differential equations (system dynamic equations and turbine governer model) and linear algebraic equations (tie-line power transfer and area generation equations), they wrote the necassary (and also sufficient under some assumptions) contidions by using Pontryagin's maximum principle. Later on, they showed that optimal controls (speed changer controls and area loads) that are obtained from the above necessary conditions can be obtained by choosing suitable real time prices (for generation and demand) by the utility companies. They formulated the real time price for generation as a linear function of frequency deviation and integral of frequency deviation. Average frequency deviation was used in their formulation. They used a demand real time price where increase in price causes decrease in demand. They ignored all losses in the system in their examples.

In our dispatch technique, our goal is to minimize the total generation cost in the system. We, too, consider average frequency deviation (average frequency error) or motion of center of inertia to model system dynamic behavior. Base spot price of electricity is also a linear function of system frequency error an integral of frequency error of the system, but we add a coefficient (A_1) into the base spot price equation. In the formulation of the problem, we suppose that all areas comprising the multi-area system has agreed upon the formulation of a base spot price in advance rather than agreeing upon a transfer schedule at a fixed price. This results in a flexible transfer schedule among them. We allow each area in the multi-area system to generate and/or exchange power to maximize its own benefit. The decision of changing generation of power in any area in the system is made according to difference between the posted spot price of electricity and its incremental generation cost of electricity. If the system losses are ignored and motion of the system's center of inertia is considered, every area in the system sees the same spot price of electricity which is the base spot price of electricity. When the system losses are neglected, using the base spot price of electricity in the dispatch algorithm is enough to find the optimum generation of each areas. As long as there is no tie-line which exceeds its transmission capacity in the system, which area buys power from which area is not important.

Inclusion of the system tie-line transmission losses are achived by modifying the base spot price of electricity. In a lossy system, knowing the rules (or algorithm) for determination of the transfer powers among areas becomes important. We propose a transfer power algorithm which uses the posted spot price of electricity of each areas in the system. Our dispatch technique can handle constraints on generations of areas and tie-line transmission capacities very easily.

2. Problem Formulation

In this section, we will demonstrate a spot price algorithm on a lossless n-area case. At the first stage, we will ignore generation and transmission tie line constraints to make the formulation easier. We will relax the simplifications one by one in a separate section.

2.1. Input-output and incremental cost curves

We assumed that generation units' input-output curves are quadratic. Mechanical inputs to the generators are taken as independent variables in those curves. An input-output curve for a generating unit is

$$M = aP_m^2 + bP_m + c \quad \text{joule/h.} \tag{1}$$

By using price per joule for used fuel $p(\text{R}/\text{joule})$[1]

$$F = Mp = (ap)P_m^2 + (bp)P_m + cp \quad \text{R}/\text{h.} \tag{2}$$

For incremental fuel cost, we obtain

$$\lambda = \frac{dF}{dP_m} = (2ap)P_m + bp \quad \text{R}/\text{MWh}$$

or

$$\lambda = BP_m + Z \quad \text{R}/\text{MWh} \tag{3}$$

where P_m has dimension of MW.

2.2. Pricing of electricity

In our algorithm all areas in multi-area system agree upon the formulation of a base spot price of electricity, SP, in advance. *The base spot price of electricity changes with the frequency error and the integral of frequency error of the system* [14]. We can write the spot price of electricity as

$$SP = A_1 - A_2\Delta f(t) - A_3 2\pi \int_0^t \Delta f(t)\, dt \quad \text{R}/\text{MWh} \tag{4}$$

where $\Delta f(t)$ represents frequency error, or deviation from the nominal frequency. When the frequency error and the integral of frequency error are positive[2], the base spot price of electricity decreases. In reverse, when they are negative[3], the base spot price of electricity increases. That is, A_2 and A_3 are positive constants. Formulation of the spot price of electricity assures system frequency regulation as long as the system stays stable. Zero time for accumulation of frequency error is the time when accumulation of frequency error or *time error* is made zero throughout the system. In Equation 4, A_1 and A_3 have dimension of R/MWh and A_2 has dimension of R/(MWh $\times$ Hz).

2.3. n-area problem

We assume that each area in the system has one generating unit (equivalent unit) [15]. Incremental generation costs of areas are represented as

$$\lambda_i = B_i P_{mi} + Z_i, \quad i = 1, ..., n \tag{5}$$

where P_{mi} is the mechanical power of the i^{th} unit and B_i and Z_i are constants.

Generating units' modified moment of inertia constants are defined as

$$\tilde{J}_i = \frac{4\pi H_i P_{i,rated}}{\omega_{syn}} = 2\pi J_i \omega_{syn}, \quad i = 1, ..., n \tag{6}$$

[1] R represents a nameless fictional monetary unit.
[2] Load decrease, system accelaration
[3] Load increase, system deceleration

where $P_{i,rated}$ is rated active power output of generating unit$-i$ and H_i is inertia constant of generating unit$-i$ defined as

$$H_i = \frac{\frac{1}{2}\omega_{syn}^2 J_i}{P_{i,rated}}, \quad i = 1, ..., n. \tag{7}$$

J_i is moment of inertia constant of generating unit$-i$ and ω_{syn} is electrical synchronous frequency in radian/sec. P_{ei} is electrical output of area$-i$. For the whole system, we make the following definitions:

$$\tilde{J} = \sum_{i=1}^{n} \tilde{J}_i \tag{8}$$

$$P_m = \sum_{i=1}^{n} P_{mi} \tag{9}$$

$$P_e = \sum_{i=1}^{n} P_{ei}. \tag{10}$$

Under the assumption that the system network is *rigid* or *strong,* we can take one frequency error for whole system. This also means that tie-lines can transmit fairly large powers. Whole system's dynamic equation can be written as

$$\Delta \dot{f}(t) = \frac{1}{\tilde{J}}(P_m - P_e) \tag{11}$$

$$\dot{\delta}(t) = 2\pi \Delta f(t) \tag{12}$$

Equations 11 and 12 represent motion of the system's center of inertia. Oscillations between rotor pairs are ignored. In the steady state, these equations give the same result obtained from the swing equation where synchronizing oscillations are not ignored [16].

Under the given pricing policy of electricity, we now discuss behavior of areas when the system load changes. Each area is going to dispatch generation to maximize its return. If one area realizes that spot price of electricity is bigger than the incremental generating cost of its generation, that area is going to increase its generation (i.e its mechanical input to its generator) until spot price of electricity is equal to its incremental generation cost. In are reverse situation, if one area sees that spot price of electricity is less than its incremental cost of its generation, it is going to decrease its generation until spot price of electricity is equal to its incremental generation cost. Under those conditions, if an area generates more than its load, it is going to seel its excess generation or if it generates less than its load, it is going to buy an amount of power which is equal to its generation deficiency. Since we ignore the system losses, every area in the system sees the same spot price which is the base spot price of electricity. It does not matter which area buys from or sells to which area in a lossless system. This dispatching policy can be expressed by the following differential equations

$$\dot{P}_{mi} = K_i(SP - \lambda_i), \quad i = 1, ..., n \tag{13}$$

where $K_1, ..., K_n$ are positive constants of dimension $(3600 \times MW^2)/\not{R}$. Dot in Equation 13 represents derivative with respect to time. For the integral of frequency error, by using Equation 12, we have

$$\delta(t) - \delta(0) = 2\pi \int_0^t \Delta f(t)\, dt. \tag{14}$$

Since at $t = 0$ frequency error accumulation is made zero, we can write

$$\delta(t) = 2\pi \int_0^t \Delta f(t)\, dt. \tag{15}$$

Time error in the system is [1]

$$\Delta Time(t) = \frac{\delta(t)}{2\pi 60} = \frac{1}{60}\int_0^t \Delta f(t)\,dt \quad \text{sec.} \tag{16}$$

$\Delta Time(t)$ in Equation 16 gives the time error of synchronous clocks in all areas. If we substitute Equations 4 and 5 into Equations 13, we obtain

$$\begin{aligned}\dot{P}_{mi}(t) \;=\; & -(K_iA_2)\Delta f(t) - (K_iB_i)P_{mi}(t) - (K_iA_3)\delta(t) \\ & +(K_iA_1 - K_iZ_i), \quad i = 1, ..., n\end{aligned} \tag{17}$$

Equation 11, 12 and 17 can be written in a compact matrix form.

$$\begin{pmatrix} \Delta\dot{f}(t) \\ \dot{P}_{m1}(t) \\ \dot{P}_{m2}(t) \\ \vdots \\ \dot{P}_{mn}(t) \\ \dot{\delta}(t) \end{pmatrix} = \begin{pmatrix} 0 & 1/\tilde{J} & 1/\tilde{J} & \cdots & \cdots & 1/\tilde{J} & 0 \\ -K_1A_2 & -K_1B_1 & 0 & \cdots & \cdots & 0 & -K_1A_3 \\ -K_2A_2 & 0 & -K_2B_2 & \cdots & \cdots & 0 & -K_2A_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -K_nA_2 & 0 & 0 & \cdots & \cdots & -K_nB_n & -K_nA_3 \\ 2\pi & 0 & 0 & \cdots & \cdots & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta f(t) \\ P_{m1}(t) \\ P_{m2}(t) \\ \vdots \\ P_{mn}(t) \\ \delta(t) \end{pmatrix} +$$

$$\begin{pmatrix} -1/\tilde{J} \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} P_e + \begin{pmatrix} 0 \\ K_1A_1 - K_1Z_1 \\ K_2A_1 - K_2Z_2 \\ \vdots \\ K_nA_1 - K_nZ_n \\ 0 \end{pmatrix} \tag{18}$$

Equation 18 is state-space representation of the considered n area system. Equation 18 can be written as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}P_e + \mathbf{k} \tag{19}$$

We want this system to be stable. That is, as $t \to \infty, P_{mi}\, i = 1, ..., n$ should go to finite values, and also $\Delta f(t)$ *must go to zero* because $\delta(t)$ goes to a constant. In this problem, $\tilde{J}$ and B_i, Z_i, $i = 1..., n$ are given parameters. By choosing coefficients of the spot price of electricity A_2, A_3 and coefficients in Equation 13 which are $K_i, i = 1, ..., n$, we can make this *linear system* stable. However, we *can not* select K_i coefficients freely. As it can be seen form Equations 13, they determine rate of change of boiler and turbine output. They have upper limits. It is much easier to make the system stable under the condition where H, B, Z and K constants are given and A_2, A_3 constants are chosen. This approach is very suitable for problems having a high dimension (more areas).

Now let us assume that we choose A_2, A_3 coefficients so that the system is stable. Since losses are ignored and a generator can match its output to its load almost instantly, we can write the following equation:

$$\Delta P_L = \Delta P_e \tag{20}$$

where ΔP_L and ΔP_e are load and generation changes in the system. Suppose that there is a load change of $\Delta P_L = \Delta P_e$ at $t = t_0^+$ and then load remains unchanged during T seconds after the load change. At $t = t_0$ the system was in steady state and its initial conditions are

$$P_{mi}(t_0) = P_{ei}(t_0), \quad i = 1, ..., n \tag{21}$$

$$\Delta f(t_0) = 0 \tag{22}$$

$$\delta(t_0) = \delta_0 \tag{23}$$

Since $P_{mi}, i = 1, ..., n$ have constant steady state values at $t = t_0$, by using Equations 13 at $t = t_0$, we can write

$$K_i(SP - \lambda_i) = 0, \quad i = 1, ..., n \tag{24}$$

Since $K_i \neq 0, i = 1, ..., n$, we have

$$\lambda_1 = \lambda_1 = \cdots \lambda_n = SP^0 = \lambda^0. \tag{25}$$

Equation 25 states that in this optimal power dispacth, the steady state results obtained here are the same as those obtained from equal incremental criteria [15]. *However, this optimal dispatch technique results in frequency regulation as well.* By using Equation 25 and Equation 4, we can write

$$A_1 - A_2\Delta f(t_0) - A_3\delta(t_0) = SP^0 \tag{26}$$

Since $\Delta f(t_0) = 0$, from Equation 26, we can write

$$A_1 = SP^0 + A_3\delta(t_0) \tag{27}$$

Since there is a load change and $\Delta P_L = \Delta P_e$ at $t = t_0^+$ and load is constant in T seconds after the load change, we can take

$$P_e^1 = P_e^0 + \Delta P_e = P_L^0 + \Delta P_L = P_L^1 = constant \tag{28}$$

where

$$\sum_{i=1}^{n} P_{ei}^0 = P_e^0 \tag{29}$$

$$\sum_{i=1}^{n} P_{Li}^0 = P_L^0 \tag{30}$$

during T seconds which is sufficiently long for the system to settle at its new steady state point. Under those conditions, there is an analytical solution to Equation 19 which is given by

$$\mathbf{x}(t) = e^{\mathbf{A}(t-t_0)}(\mathbf{x}(t_0) + \mathbf{A}^{-1}\mathbf{c}) - \mathbf{A}^{-1}\mathbf{c} \tag{31}$$

where

$$\mathbf{x}(t_0) = \begin{pmatrix} 0 \\ P_{m1}(t_0) \\ \vdots \\ P_{mn}(t_0) \\ \delta(t_0) \end{pmatrix} \tag{32}$$

and

$$\mathbf{c} = \mathbf{b}P_e^1 + \mathbf{k} \tag{33}$$

The solution can also be given in terms of system's eigen values

$$\mathbf{x}(t) = \sum_{i=1}^{n+2}[e^{\nu_i(t-t_0)}\mathbf{R}_i(\mathbf{x}(t_0) + \mathbf{A}^{-1}\mathbf{c})] - \mathbf{A}^{-1}\mathbf{c} \tag{34}$$

where $\mathbf{R}_i$ is the residue matrix corresponding to eigen value ν_i. As it is seen from Equation 34, as t, which is bigger than t_0, increases, transient terms will die out and the system will go to a new steady state point given as $-\mathbf{A}^{-1}\mathbf{c}$. At the new steady state point at $t = t_1$, where $(t_0 + T) \geq t_1$, we have (asymptotically)

$$\mathbf{x}(t_1) = -\mathbf{A}^{-1}\mathbf{c}. \tag{35}$$

Since again at $t = t_1, P_{mi}, i = 1, ..., n$ have constant values, we have

$$\lambda_1 = \lambda_2 = \cdots = \lambda_n = SP^1 = \lambda^1 \tag{36}$$

This result show us that at new steady state point, load dispatch is optimum again. Also because of pricing strategy used, we have at $t = t_1$

$$\sum_{i=1}^{n} P_{mi}(t_1) = \sum_{i=1}^{n} P_{ei}(t_0) + \Delta P_e = P_L^1 \tag{37}$$

Since $\Delta f(t_0) = 0$ initially, by using Equation 11 we have[4]

$$\Delta f(t_1) = 0 \tag{38}$$

This very important result expresses that at any new steady state point, the frequency error is zero. We see from these results that by using this base spot price of electricity for interchange of electricity, each area will control its generation to maximize its return. If each area in the system controls its generation to maximize its return, the generation obtained gives a global optimum dispatch for the system. A natural question which can be asked at this point is that what happens if one of the areas or more do not control its generation to maximize its return. This question can be discussed under constrained generation case where controlling of generation in non-optimum fashion can be taken as putting an artificial constraint on that area's generation.

2.4. Constrained generation case

If the imposed constraints on area generations are given as

$$P_{mi_{min}} \leq P_{mi} \leq P_{mi_{max}} \quad i = 1, ..., n \tag{39}$$

where $P_{mi_{min}}$ and $P_{mi_{max}}$ terms are maximum and minimum generation limits of area$-i$ respectively, then the dynamic Equation 17 must be rewritten as

$$\dot{P}_{mi} = \begin{cases} 0 & \text{if } P_{mi} = P_{mi_{max}} \text{ and } \dot{P}_{mi} > 0 \text{ or } P_{mi} = P_{mi_{min}} \text{ and } \dot{P}_{mi} < 0 \\ \dot{P}_{mi} & \text{otherwise} \end{cases} \tag{40}$$

where $\dot{P}_{mi}$ is given in Equation 17. These n equations, along with Equations 11 and 12 become the dynamic equations of the system. That is, the rate of change of mechanical power of the i^{th} generator has the same formula as in the unconstrained case, unless the generation of that unit is at its maximum or minimum and the usual rule says to increase or decrease generation respectively. If generation is at a maximum or minimum and old rules would ask for still greater or lower generation, the generation would stay at its upper or lower limit respectively. If, for instance, generator 1 is at its maximum limit, P_{m1} becomes a constant in Equation 11 and the system behaves as a $n-1$ generator system. This lower order system should still reach an equilibrium at rated frequency (zero frequency error) and global optimal power distribution subject to

[4] Consider also equation 37

the generation constraint. The system incremental generation cost and the base spot price would be greater than those without a generation constraint, as would the integral of frequency error.

If $n-1$ of the generators reach the maximum generation limits, the same arguments apply. The n area problem reduces to a single area problem, with the remaining area controlling the frequency. If all areas reach their maximum generation limit, the system will collapse and load shedding will be required.

2.5. Inclusion of the tie-line transmisson losses

When the tie-line transmission lossesy are included in the dispatch technique, the transfer power among areas become important. Therefore, we will develop a power transfer algorithm for lossy case. The governing differential set in a lossy case can be written as

$$\Delta \dot{f} = \frac{1}{\tilde{J}}(P_m - P_L - P_{loss}) \tag{41}$$

$$\dot{P}_{mi} = K_i(SP_i - \lambda_i), \quad i = 1, ..., n \tag{42}$$

$$\dot{\delta} = 2\pi \Delta f \tag{43}$$

where

$$SP_i = \theta_i SP \quad i = 1, ..., n \tag{44}$$

and $SP, \lambda_i, \tilde{J}$ and P_m are given in Equation 4,5, 8 and 9 respectively. P_L represents sum of all area loads. If Equation 5 and 44 are substituted into Equation 42, we obtain

$$\begin{aligned} \dot{P}_{mi}(t) \; = \; & -(K_i\theta_i A_2)\Delta f(t) - (K_i B_i)P_{mi}(t) - (K_i\theta_i A_3)\delta(t) \\ & +(K_i\theta_i A_1 - K_i Z_i), \quad i = 1, ..., n. \end{aligned} \tag{45}$$

If we rewrite Equation 41, 45 and 43 in matrix format, we get

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{d} \tag{46}$$

where

$$A = \begin{pmatrix} 0 & 1/\tilde{J} & 1/\tilde{J} & \cdots & \cdots & 1/\tilde{J} & 0 \\ -K_1\theta_1 A_2 & -K_1 B_1 & 0 & \cdots & \cdots & 0 & -K_1\theta_1 A_3 \\ -K_2\theta_2 A_2 & 0 & -K_2 B_2 & \cdots & \cdots & 0 & -K_2\theta_2 A_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -K_n\theta_n A_2 & 0 & 0 & \cdots & \cdots & -K_n B_n & -K_n\theta_n A_3 \\ 2\pi & 0 & 0 & \cdots & \cdots & 0 & 0 \end{pmatrix} \tag{47}$$

$$\mathbf{d} = \begin{pmatrix} \frac{1}{\tilde{J}}(-\sum_{i=1}^{n} P_{Li} - P_{loss} \\ K_1(\theta_1 A_1 - Z_1) \\ K_2(\theta_2 A_1 - Z_2) \\ \vdots \\ K_n(\theta_n A_1 - Z_n) \\ 0 \end{pmatrix}. \tag{48}$$

$\dot{\mathbf{x}}$ and $\mathbf{x}$ are the same as those given in Equation 18. In the case of constrained generation, Equation 40 will also be valid, but the value of $\dot{P}_{mi}$ should be calculated according to Equation 45. For given K_i, B_i, Z_i and $\theta_i = 1\,(i = 1, ..., n)$, we can make the system stable by choosing suitable A_2 and A_3 values. Calculation of θ_i values is given in the following section.

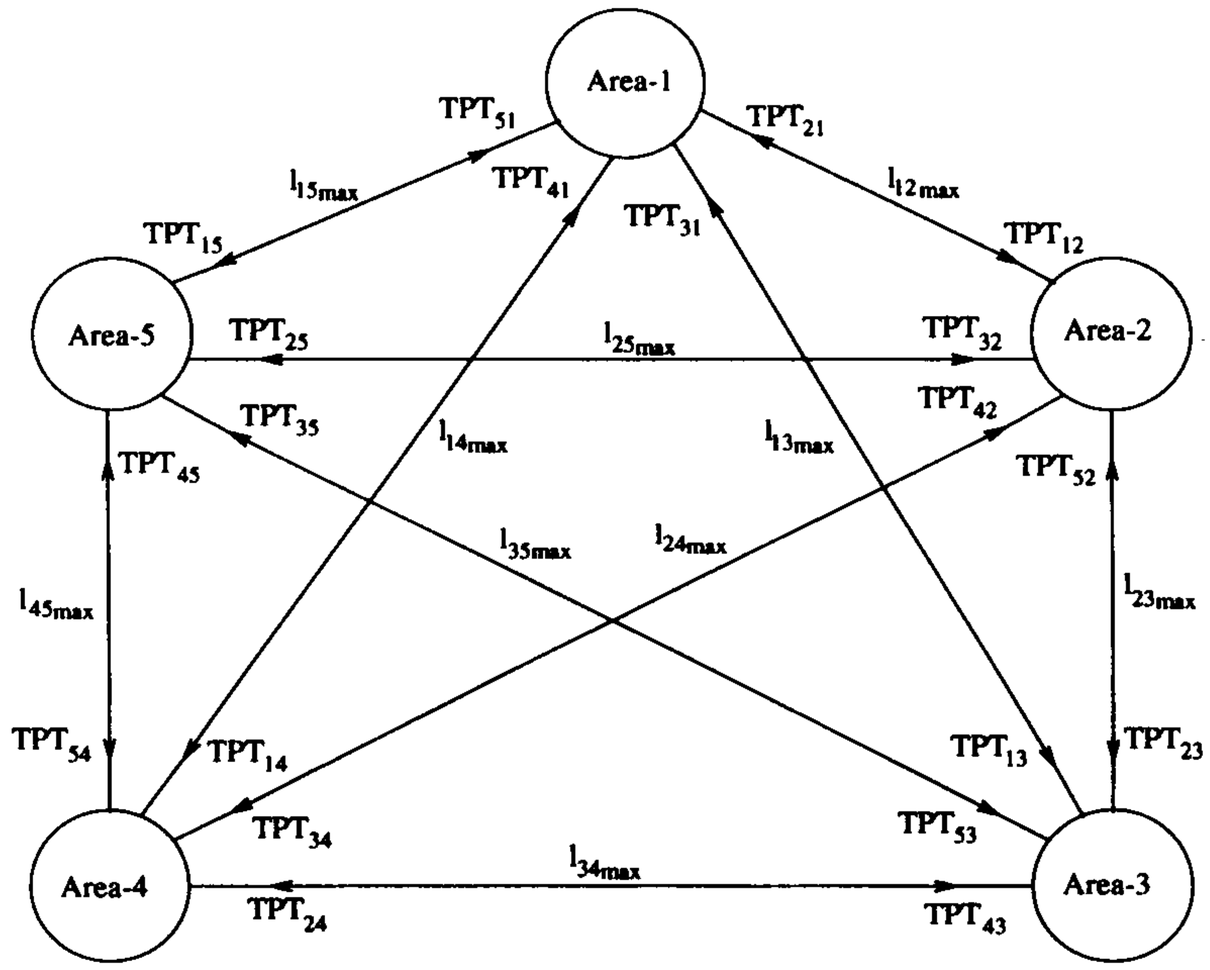


Figure 1. Representation of the most general five area system.

3. Formulation of Transfer Power Algorithm

Tie-line power flows depend on the tie-line configuration of the considered system. Therefore, we will consider the most general five area $(n = 5)$ case as an example here. Derivation procedure which is given here can be extended to the cases where there are more than five areas in the system. representsation of the most general five area system is shown in Figure 1 where every area is connected to every area by a tie-line. $l_{ij_{max}} MW$ represent maximum transmission capacity of tie-line $ij, i = 1, ..., 5 \;\; j = i + 1, ..., 5$ as shown in Figure 1. To determine whether tie-line transmission capacities are exceeded or not, we need to know *the transfer powers* among areas during transient and the steady state periods. During the transient period, there is no generation-load balance in the overall system, and frequency change occurs. For example, if there is a load increase in the system, the system's frequency decreases. Decrease of frequency is stopped at the point where load and generation are equal system wide. During this deceleration period generation is less than load. When a customer applies an electrical load, it receives necessary power immediately although mechanical inputs can not be changed immediately. This power comes from stored kinetic energy on rotating masses of the system. If there is a load change in the system, kinetic powers assigned to each area will be

$$P_{kin,i} = \frac{\tilde{J}_i}{\tilde{J}}(P_m - P_L - P_{loss}), \quad i = 1, ..., 5 \tag{49}$$

As it is seen from Equation 49, every area gives (or gains) kinetic power according to its modified moment of inertia constant.

In the transient period, mechanical energy is converted to electrical energy and also stored as kinetic energy on the rotating mass of the system. The mechanical power of the i^{th} generator is taken

$$P_{mi}(t) = P_{ei}(t) + P_{kin,i}(t), \quad i = 1, ..., 5 \tag{50}$$

We are concerned with the transferable power of an area which is the generated power minus load of an area

rather than total input power here. So, for transfer powers, we have

$$PT_i = P_{ei} - P_{Li} = P_{mi} - P_{kin,i} - P_{Li}, \quad i = 1, ..., 5 \tag{51}$$

where P_{Li} is load power of area$-i$. Note that $P_{kin,i}(t)$ becomes zero in steady state. Therefore Equation 51 is also valid for steady state. When $PT_i > 0$, area$-i$ has generation excess and when $PT_i < 0$, area$-i$ has generation deficiency. We will assume that every area in the system has agreed upon the following two conditions:

- Any area can buy power from any area at the seller area's posted spot price.
- None of the areas can force any area to buy power from itself.

There are four kinds of area types in view of sold and bought power.

Absolute seller area: the area only sells, never buys any power.

Absolute boyer area: the area only buys, never sells any power.

Net seller area: the area sells and buys power, but it sells more power than it buys.

Net buyer area: the area sells and buys power, but it buys more power than it sells.

Net power of area$-i$ is described as

$$P_{net_i} = SELL_i - BUY_i \tag{52}$$

where $SELL_i$ and BUY_i represent power sold and bought by area$-i$. According to the given definitions, both absolute buyer area and net buyer area have *negative* net power. On the other hand, both absolute seller and net seller area have *positive* net power. *An important point here is that a system should not consist of only net seller and net buyer areas in the steady state since there must be at least one area in the system which buys its own sold power in this case.* In the following sections, we will develop a transfer power algorithm step by step. Before starting the development of the transfer power algorithm, we will shed some light on tie-line power loss calculation. When there is a power flow from area-i to area-j, loss expression for tie-line ij can be written as

$$P_{loss_{ij}} = C_{ij}(TPT_{ij})^2 R_{ij}. \tag{53}$$

In Equation 53, R_{ij} is the resistance of tie-line ij (ohm) and C_{ij} is defined as follows

$$C_{ij} \triangleq \frac{10^6}{(V_{ij}\cos\Phi_{ij})^2} \tag{54}$$

where V_{ij} and Φ_{ij} are transmission voltage (volt) and power factor of tie-line ij on the border of area$-j$ side. *From now on, transfer power TPT_{ij} is going to represent transfer power of tie-line ij on area-j border.* Of course, transfer power of tie-line ij on area$-i$ border is

$$-TPT_{ji} = TPT_{ij} + P_{loss_{ij}} \tag{55}$$

For the sake of simplicity, we will assume constant V_{ij} and Φ_{ij} along tie-line ij. With this assumption, we can take $C_{ij} = C_{ji}$ in our simulation program. In Equation 53, TPT_{ij} is put as MW and obtained loss has dimension of MW also.

3.1. Buying policy of an area

We will explain buying policy of area-1 merely as an example which is also valid for other areas in the system. Every area in the system tries to buy the cheapest power in the transfer power algorithm. Let us assume that at $t = t_0, SP_1^0, SP_2^0, SP_3^0, SP_4^0$ and SP_5^0 represent posted spot price of area-1 through area-5. Let us

also suppose that area-1 needs to buy a specific amount of power which is represented as BUY_1. Area-1 will try to buy a certain amount of power from each remaining four areas such that the total bought power will add up to BUY_1 and the cost of bought power will be minimum. *The cost of the bought power becomes minimum only when area-1 sees the same spot price of electricity on tie-lines connecting area-1 and other remaining areas at its border.* The above proposition can be proved by assuming that spot prices on tie-lines seen by area-1 are *not* equal. We accepted that spot prices of electricity seen by area-1 on tie-lines at its border increases proportional to corresponding losses taking place during transmission of the corresponding transfer powers. By using the above optimal buying policy, we can calculate tie-line transfer powers at the area-1 border. These tie-line transfer powers are $TPT_{21}, TPT_{31}, TPT_{41}$ and TPT_{51} (see Figure 1). The spot price of electricity seen by area-1 on tie-line 12 at its border is

$$SP_{1_{(2)}} = (1 + \frac{C_{12}(TPT_{21})^2 R_{12}}{TPT_{21}})SP_2^0 = (1 + C_{12}TPT_{21}R_{12})SP_2^0. \tag{56}$$

Other spot price of electricity values seen by area-1 on tie-line 13 through tie-lien 15 can be calculated in a similar manner. According to the optimal buying policy of area-1, we have

$$SP_{1_{(2)}} = SP_{1_{(3)}} = SP_{1_{(4)}} = SP_{1_{(5)}} \tag{57}$$

$$TPT_{21} + TPT_{31} + TPT_{41} + TPT_{51} = BUY_1 \tag{58}$$

By using explicit expressions for the spot prices, we can rewrite Equation 57 as

$$(1 + C_{12}TPT_{21}R_{12})SP_2^0 = (1 + C_{13}TPT_{31}R_{13})SP_3^0 =$$

$$(1 + C_{14}TPT_{41}R_{14})SP_4^0 = (1 + C_{15}TPT_{51}R_{15})SP_5^0 \tag{59}$$

Equation 59 gives three independent algebraic equations and the necessary fourth equation comes from Equation 58. By using four independent equations, four unknown tie-line powers can be calculated. If all calculated tie-line transfer powers are positive, area-1's posted "back door" spot price will be

$$SP_1 = (1 + C_{12}TPT_{12}R_{12})SP_2^0 \tag{60}$$

There may be a situation where some of the calculated bought powers can be negative. As stated before, buying negative power means selling positive power and since none of the areas in the system can force any other area to buy power from itself, *we will take negative tie-line transfer powers as zero* in the calculation of bought power from other areas. If, for example, TPT_{31} as found to be negative, it is taken as equal to zero and the remaining tie-line transfer powers are calculated by using the following equations.

$$(1 + C_{12}TPT_{21}R_{12})SP_2^0 = (1 + C_{14}TPT_{41}R_{14})SP_4^0 =$$

$$(1 + C_{15}TPT_{51}R_{15})SP_5^0 \tag{61}$$

$$TPT_{21} + TPT_{41} + TPT_{51} = BUY_1 \tag{62}$$

Again at this point, if there are negative calculated tie-line transfer powers, they are taken as zero and described calculation procedure continues until all calculated tie-line transfer powers positive. Optimal buying policies of other areas are exactly the same as that of area-1.

3.1.1. Handling different system configurations

If there is no-tie-line between two areas, the absent tie-line is not considered in the transfer power calculation in the areas located at the each end of the absent tie-line. For example, if tie-line 12 does not exist, it is not taken into consideration in calculation of tie-line transfer powers of area-1 and area-2. Therefore, we get $TPT_{21} = TPT_{12} = 0\,MW$ from the calculations. In the case of two and three tie-lines not existing, the same logic is applied. If four tie-lines which connect an area to the rest of four areas do not exist, the considered area becomes isolated. In this situation, the simulation program gives a warning to the user. Our simulation program does not handle such situations.

3.2. Calculation of bought power of an area

When the net power of an area is equal to its transferable power, that area will be called a *balanced* area. In steady state, all areas in the system must be balanced. In other words

$$PT_i = P_{net_i} \quad i = 1, ..., 5. \tag{63}$$

where PT_i and P_{net_i} are given in Equation 51 and 52 respectively. If one area is not balanced, its state will satisfy one of the conditions given below. Again wel will take area-1 as an example here. Conditions for other areas can be obtained just by changing subscripts. Let us rewrite equations for P_{net_1} and PT_1.

$$P_{net_1} = SELL_1 - BUY_1$$

$$PT_1 = P_{m1} - P_{L1} - P_{kin,1}$$

Possible unbalanced conditions are given below.

I) $PT_1 > 0$ and $P_{net_1} > 0$

There are two possible unbalanced cases here.

a) $P_{net_1} > PT_1$

This condition states that area-1 is selling more than its excess generation. Since area-1 has to sell that amount of power, in other words, sold power of area-1 ($SELL_1$) is not controlled by area-1, it has to buy more power to make itself a balanced area. The new bought power of area-1 is

$$BUY_1 = BUY_1^0 + (P_{net_1} - PT_1) \tag{64}$$

where BUY_1^0 is the previous bought power of area-1

b) $P_{net_1} < PT_1$

This condition says that area-1 is selling less than what it is supposed to sell. But since area-1 can not control its sold power, it can only decrease its bought power if it is buying any power. Its new bought power will be

$$BUY_1 = BUY_1^0 + (P_{net_1} - PT_1). \tag{65}$$

If the bought power of area-1 becomes negative, it should be taken as zero since negative bought power is positive sold power. If BUY_1 is taken as zero MW, area-1 is not going to be in a balanced condition after having zero bought power. If an area is not buying any power from any area, its posted spot price should be the base spot price of the system. Since rapid changes in the posted spot price can cause *oscillations* in our transfer power algorithm, we do not change the spot price of any area whose situation looks like the one

given in the above, quickly to the base spot price of the system; instead, we make gradual decreases towards the base spot price[5]. Area-1 becomes an absolute seller area after making its bought power zero.

II) $PT_1 < 0$ and $P_{net_1} < 0$

There are two unbalanced cases under this condition.

a) $|P_{net_1}| > |PT_1|$

This condition states that area-1 is buying more than its need. Since it can not force another area to buy from itself, it decreases its bought power. Its new bought power will be

$$BUY_1 = BUY_1^0 + (|PT_1| - |P_{net_1}|). \tag{66}$$

If its new bought power becomes negative, it is taken as zero.

b) $|PT_1| > |P_{net_1}|$

The condition given above says that area-1 is buying less than it is supposed to buy. Therefore it has to buy more. Its new bought power is

$$BUY_1 = BUY_1^0 + (|PT_1| - |P_{net_1}|). \tag{67}$$

III) $PT_1 > 0$ and $P_{net_1} < 0$

This condition states that area-1 is buying too much power, therefore it has to decrease the amount of power purchased. If its bought power becomes negative, it has to be taken as zero. Its new bought power is

$$BUY_1 = BUY_1^0 + (P_{net_1} - PT_1). \tag{68}$$

IV) $PT_1 < 0$ and $P_{net_1} > 0$

The condition given above expresses that area-1 is selling too much power, therefore it has to buy more. Its new bought power is

$$BUY_1 = BUY_1^0 + (P_{net_1} + |PT_1|). \tag{69}$$

3.3. Arbitration of power purchases between two areas

We will consider only tie-line 12 connecting area-1 and area-2 as an example in this section. Necessary computation for other tie-lines are exactly the same as the one which is given here.

As explained in section 3.1, the tie-line transfer powers calculated at each area's border (see Figure 1) are either *positive numbers or zero.* Possible relations of tie-line transfer powers calculated at each end of tie-line 12 can be classified in three main cases. These cases are given below.

1) $TPT_{12} > TPT_{21}$

We will represent new tie-line transfer powers as $CTPT_{ij}$ after arbitration of power purchases. New tie-line transfer powers at each area's border will be

$$CTPT_{12} = TPT_{12} - TPT_{21} \tag{70}$$

$$CTPT_{21} = -(CTPT_{12} + C_{12}(CTPT_{12})^2 R_{12}) \tag{71}$$

2) $TPT_{12} < TPT_{21}$

New tie-line transfer powers at each area's border will be

$$CTPT_{21} = TPT_{21} - TPT_{12} \tag{72}$$

[5] Look at section 3.4 for details

$$CTPT_{12} = -(CTPT_{21} + C_{12}(CTPT_{21})^2 R_{12}) \tag{73}$$

3) $TPT_{12} = TPT_{21}$

New transfer powers will be

$$CTPT_{12} = CTPT_{21} = 0 \tag{74}$$

According to the optimal buying policy, every area calculates their tie-line transfer powers which go into respective areas. Although those calculated tie-line powers of an area are nonnegative, some of them can become negative after the power purchase arbitration procedure. The sum of negative tie-line powers going into an area gives the power sold by a considered area. The sum of positive tie-line powers going into an area gives the power bought by a considered area, so we can write the following equations (for area-1)

$$SELL_1 = \sum_{j \in \{TPT_{j1} < 0\}} |TPT_{j1}| \tag{75}$$

$$BUY_1 = \sum_{j \in \{TPT_{j1} > 0\}} TPT_{j1} \tag{76}$$

As long as both calculated tie-line powers TPT_{12} and TPT_{21} are different from zero, both area-1 and area-2 are going to be unbalanced after the power purchase arbitration. Therefore, in a solution point where all areas in the system become balanced, smaller calculated tie-line power (in the case of equal calculated tie-line powers as in case-3, both tie-line powers) should be zero for every tie-line before the arbitration of bought powers.

3.3.1. Handling of constrained tie-line transmission

There can only be a tie-line transmission constraint hit in case-1 and case-2 in section 3.3. Again we will take tie-line 12 as an example here. In case-1 where $TPT_{12} > TPT_{21}$, the absolute value of $CTPT_{21}$ should be less than or equal to $l_{12_{max}}$ value. If $|CTPT_{21}|$ is found to be larger than $l_{12_{max}}$, it should be taken as equal to $l_{12_{max}}$. When $|CTPT_{21}|$ is equal to $l_{12_{max}}$ value, $CTPT_{12}$ should also get its maximum value. Maximum $CTPT_{12}$ value is found by using the following equality

$$|CTPT_{21}|_{max} = C_{12}(CTPT_{12})^2 R_{12} + CTPT_{12} = l_{12_{max}}$$

or

$$C_{12}(CTPT_{12})^2 R_{12} + CTPT_{12} - l_{12_{max}} = 0 \tag{77}$$

quadratic polynomial as

$$CTPT_{12_{max}} = \frac{-1 + \sqrt{1 + 4R_{12}C_{12}l_{12_{max}}}}{2C_{12}R_{12}}. \tag{78}$$

Maximum allowable values of $CTPT_{12}$ and $CTPT_{21}$ for case-2 in section 3.3 can be found by following the same reasoning given above. Their values are found to be

$$CTPT_{21_{max}} = \frac{-1 + \sqrt{1 + 4C_{12}R_{12}l_{12_{max}}}}{2C_{12}R_{12}} \tag{79}$$

$$|CTPT_{12}|_{max} = l_{12_{max}}. \tag{80}$$

If the transfer powers of $CTPT_{12}$ and $CTPT_{21}$ found in case-1 and case-2 of section 3.3 are larger than their respective maximum allowable limits, they are taken as their respective maximum allowable limits. But unless the buyer area[6] is discouraged to buy over tie-line 12, it will continue to want to buy more power

[6] Buyer area is area-2 and seller area is area-1 in case-1. Buyer area is area-1 and seller area is area-2 in case-2.

over tie-line 12, and eventually both area-1 and area-2 will stay unbalanced. Therefore the solution point for given transferable powers of areas will not be reached. To discourage the buyer area which causes the tie-line 12 transmission constraint hit, we make the buyer area see a spotprice of electricity on tie-line 12 which increases proportional to the difference between found transfer power after arbitration and maximum tie-line capacity of tie-line 12. To do this job, we increase resistance value of tie-line 12 artifically according to the explained rule. For example, if $CTPT_{12}$ exceeds $CTPT_{12_{max}}$ or $|CTPT_{21}|$ exceeds $l_{12_{max}}$ artificially increased resistance value of tie-line 12 is

$$R_{12_{iner}} = \left(1 + \gamma \frac{|CTPT_{21}| - l_{12_{max}}}{l_{12_{max}}}\right) R_{12}\,, \quad |CTPT_{21}| > l_{12_{max}}. \tag{81}$$

The same resistance value for case-2, when there is tie-line hit, is

$$R_{12_{iner}} = \left(1 + \gamma \frac{|CTPT_{12}| - l_{12_{max}}}{l_{12_{max}}}\right) R_{12}\,, \quad |CTPT_{12}| > l_{12_{max}}. \tag{82}$$

where γ is *convergence factor*. The reason for using of γ will be explained in section 3.4.

3.4. Overall transfer power algorithm

Transfer power algorithm is used for both initial (or steady state) and transient transfer power calculations. It uses area power buying policy subroutine and power purchase arbitration subroutine.

Each area power buying policy subroutine gets other areas' posted "back door" spot prices ($SP_j^0, j = 1, ..., 5, j \neq i$) the necessary bought power of the considered area (BUY_i) and calculates (outputs) transfer powers at the considered area's border ($TPT_{ji},\ j = 1, ..., 5, j \neq i$), and new pasted spot price of the considered area (SP_i).

The power purchase arbitration subroutine receives the transfer powers calculated at the both ends of a tie-line, tie-line's resistance value, the convergence factor (γ), and outputs new transfer powers and artificially increased value of the tie-line resistance if it is necessary.

We also wrote the transfer power algorithm as a subroutine. It receives every area's posted "back door" spot price and mechanical input, the system's base spot price and total tie-line loss (in the case of initial value calculation), tolerance values for each areas balance condition $\varepsilon_i, i = 1, ..., 5$), the convergence factor (γ) and outputs new tie-line transfer powers, the posted "back door" spot price of each area and the system's total tie-line loss. It finds the transfer powers among areas such that absolute value of the difference between each area's transferable power, and net power becomes less than the respective area's tolerance value for its balance condition.

There are two important points in the transfer power subroutine which should be explained. The first one is that if all transfer powers going into an area are negative, a new posted "back door" spot price of the area is calculated according to

$$SP_i = \left(1 + \frac{P_{net_i} - PT_i}{PT_i}\right) SP_i^0 \tag{83}$$

If an area has only transfer powers which are going into that area and have negative values, that area only sells power (absolute seller area). When the net power of an absolute seller area is greater than its transferable power, it needs to buy more power and eventually increase its posted spot price. But since this is an absolute seller area, it does not buy any power, so it increases its posted spot price according to Equation 83 to discourage buyer areas and eventually reach its balance condition. If an area is an absolute seller area and its transferable power is greater than its net power, that area needs to sell more power. Therefore it should encourage other areas to buy power from itself by decreasing its posted spot price. In fact for the

condition where $P_{net_i} < PT_i, SP_i$ decreases in Equation 83. But it should not be less than the system base spot price (SP). If it is so, it is taken as equal to SP. The second point is that the new posted spot prices for the next iteration inside the power transfer subroutine are calculated according to

$$SP_i^0 = SP_i^0 + \gamma(SP_i - SP_i^0) \quad i = 1, ..., 5 \tag{84}$$

SP_i^0 which is on the right hand side of Equation 84 is the posted spot price of area$-i$ at the beginning of the current iteration. SP_i is the spot price which is obtained inside area$-i$ buying policy subroutine or by using Equation 83 at the end of the current iteration. γ is *convergence factor* and $\gamma \leq 1$. Since the obtained transfer powers inside the area buying policy subroutines are *very sensitive* to the posted spot prices of areas, large changes in the spot prices make large changes in transfer powers, eventually causing *oscillations* around the solution point. For that reason, we use a convergence factor, γ, in the final posted spot price calculation.

4. Initial or Steady State Value Calculation

Before explaining the whole calculation procedure, we will first explain the calculation of steady state values of $P_{mi}(i = 1, ..., 5)$, for the given $\theta_i(i = 1, ..., 5)$, P_{loss} and P_L values.

In a steady state, we have

$$\lambda_i = \theta_i SP \quad i = 1, ..., 5 \tag{85}$$

or

$$\frac{\lambda_1}{\theta_1} = \frac{\lambda_2}{\theta_2} = \frac{\lambda_3}{\theta_3} = \frac{\lambda_4}{\theta_4} = \frac{\lambda_5}{\theta_5} = SP. \tag{86}$$

Since the sum of the mechanical power must be equal to the system load plus the total system tie-line loss in a steady state point, we can write.

$$\sum_{i=1}^{5} P_{mi} = P_L + P_{loss}. \tag{87}$$

Equation 86 gives four independent algebraic equations. The necessary fifth equation to find five unknown values, which are $P_{mi}(i = 1, ..., 5)$, comes from Equation 87. As stated before, the initial value calculation procedure can handle the constrained generation and tie-line transmission. The constrained tie-line transmission is handled inside the transfer power subroutine during the power purchase arbitration procedure. We will now explain the calculation procedure for the constrained generation. If all obtained mechanical inputs are less than or equal to their corresponding maximum generation limits $(P_{mi_{max}}, i = 1, ..., 5)$, no further calculation is needed. But if one or more obtained mechanical inputs are larger than their maximum limits, their values are taken as their maximum limits and the remaining load is divided among the other areas optimally. For example, let us suppose that the calculated P_{m1} and P_{m2} values are larger than $P_{m1_{max}}$ and $P_{m2_{max}}$ respectively. P_{m1} and P_{m2} are taken as equal to $P_{m1_{max}}$ and $P_{m2_{max}}$ respectively. The remaining mechanical input values are calculated by using the following equations

$$\frac{\lambda_3}{\theta_3} = \frac{\lambda_4}{\theta_4} = \frac{\lambda_5}{\theta_5} = SP \tag{88}$$

$$\sum_{i=3} P_{mi} = P_L + P_{loss} - P_{m1_{max}} - P_{m2_{max}}. \tag{89}$$

The last calculated mechanical powers (P_{m3}, P_{m4}, P_{m5}), are checked against their maximum generation limits. If some of them are larger than their maximum generation limits, they are taken as equal to their

maximum generation limits. If all calculated mechanical input powers hit their maximum generation limits at the same time, there is no solution to the problem for the given area loads and maximum generation limits. Load shedding is required.

We put all related calculation for area mechanical inputs into a subroutine. It receives P_L, θ_i and P_{loss} and calculates (outputs) P_{mi} values. Initial value calculation subroutine uses area mechanical input calculation and transfer power subroutines.

The initial value calculation problem can be stated as follows: Given the area loads$(P_{Li},\ i = 1, ..., 5)$ find each area's generation $(P_{mi},\ i = 1, ..., 5)$, the transfer powers among areas $(TPT_{ij}, i = 1, ..., 5, j = 1, ..., 5, i \neq j)$ such that the cost of generated power all over the system becomes minimum. Since the transfer powers among areas are not known initially, the posted area spot prices (and θ_i values) and the total tie-line losses can not be known. Therefore we take $\theta_i = 1(i = 1, ..., 5), P_{loss} = 0$ initially. Since we have taken the initial total tie-line loss value to be zero, and the tie-line loss values are considered in the power purchase arbitration procedure[7], the difference between area transferable powers and area net powers is going to be fairly large. Therefore, the initial ε_i values must be comparably large. If they are not taken large enough, the stopping criteria of the transfer power subroutine (see section 3.4) is not going to be satisfied, so endless iteration inside the transfer power subroutine will take place. As the iteration continues inside the initial value calculation subroutine, each newly found $P_{mi}, \theta_i, SP_i, SP, P_{loss}$ and TPT_{ij} values will be closer to their respective solution point values. Therefore we decrease the ε_i values more and more as the iteration proceeds inside the initial value calculation subroutine. At the end, all of the ε_i values become less than the prescribed tolerance value.

5. Simulation Program

The algorithm of the simulation program is given in the following.

Step 1: Get $P_{Li}, i = 1, ..., 5$ values.

Step 2: Calculate initial values for $P_{mi}, SP_i, TPT_{ij}, SP$ and $P_{loss}, i = 1, ..., 5, j = 1, ..., 5, i \neq j$ (Initial value calculation subroutine).

Step 3: Set $t_1 = 0, \Delta f = 0, \delta = 0$

Step 4: Store values of $P_{mi}, SP_i, TPT_{ij}, SP, \Delta f$ and $\Delta Time$ for plotting $i = 1, ..., 5, j = 1, ..., 5, i \neq j$.

Step 5: Calculate $A_1 = SP + A_2\Delta f + A_3\delta$

Step 6: Get area load changes, ΔP_{Li}, and calculate new area loads, $P_{Li} = P_{Li} + \Delta P_{Li}, i = 1, ..., 5$. Set $t = 0$.

Step 7: Get values for ε_1 and $\gamma, i = 1, ..., 5$.

Step 8: Set $t = t + t_{incr}$. Find solution to the governing differential equation set at time t by using initial conditions at time $t - t_{incr}$.

Step 9: If $P_{mi} > P_{mi_{max}}, i = 1, ..., 5$ then $P_{mi} = P_{mi_{max}}$.

Step 10: Calculate $SP = A_1 - A_2\Delta f - A_3\delta, SP_i^0 = \theta_i SP$.

Step 11: Calculate SP_i, θ_i, TPT_{ij} and $P_{loss}, i = 1, ..., 5, j = 1, ..., 5, i \neq j$ (Transfer power subroutine).

Step 12: Set $t_1 = t_1 + t_{incr}$. Store $P_{mi}, SP_i, TPT_{ij}, SP, \Delta f$ and $\Delta Time$ for plotting, $i = 1, ..., 5$, $j = 1, ..., 5$, $i \neq j$.

Step 13: Take the current solution to the governing differential equation set as the initial conditions for the next solution step.

[7] see section 3.3

Step 14: If $t < t_{final}$ then goto step 8.

Step 15: If there is another load change then goto step 6.

Step 16: If there is another loading to the system then goto step 1.

Step 17: Stop.

The first stage in the simulation program is the calculation of the initial values of $P_{mi}, SP, SP_i, \theta_i, P_{loss}, TP$ for the given area loads. After that, area load changes are given and the values of $\Delta f, \delta, \Delta Time, SP, SP_i, P_{mi}, TP$ are found and stored at points which are apart from each other by t_{incr} untill the another loading is given to the system.

Solution to the governing differential equation set is calculated by using

$$\mathbf{x}(t) = e^{\mathbf{A}(t-t_0)}(\mathbf{x}_0 + \mathbf{A}^{-1}\mathbf{d}) - \mathbf{A}^{-1}\mathbf{d}. \tag{90}$$

The matrix $\mathbf{A}$ and the vector $\mathbf{d}$ are given is Equations 47 and 48 respectively. Since θ_i and P_{loss} values change from one solution step to another before reaching the steady state point in the transient value calculation, elements of the matrix $\mathbf{A}$, and the vector $\mathbf{d}$ also change. We assume that power steering in the system is done very fast compared to mechanical input changes.

We used time variable t_1 just for plotting purpose in the simulation program. Although time variable t is made zero after each load change, t_1 is made zero at the beginning of each new problem.

6. Numerical Example

We picked incremental generation cost coefficients for five areas as $B_1 = 0.1, B_2 = 0.15, B_3 = 0.2, B_4 = 0.25, B_5 = 0.3$ ₤$/(MW^2h)$ and $Z_1 = 1.0, Z_2 = 2.0, Z_3 = 1.0, Z_4 = 1.5, Z_5 = 2.0$ ₤$/MWh$. It is seen that the most efficient area is area-1 and the least efficient area is area-5. The efficiencies are in decreasing order.

The K_i coefficients are chosen as $K_1 = 3.0, K_2 = 2.5, K_3 = 2.0, K_4 = 1.5$ and $K_5 = 1.0\,(3600) \times MW^2)/$ ₤. The $\tilde{J}_i$ constants are selected as $\tilde{J}_1 = 350\ 10^6, \tilde{J}_2 = 300\ 10^6, \tilde{J}_3 = 275\ 10^6, \tilde{J}_4 = 250\ 10^6$ and $\tilde{J}_5 = 225\ 10^6 (kg \times m^2)/sec$.

For the given $B_i, K_i, \tilde{J}_i$ constants, we chose A_2 and A_3 coefficients of the system's base spot price as 10.0 ₤$/(MWh \times Hz)$ and 1.0 ₤$/MWh$ respectively. The eigen values of the system, when all loss in the system is ignored (i.e. $\theta_i = 1.0, i = 1, ..., 5$), are found to be $\nu_1 = -0.39378, \nu_2 = -0.37500, \nu_3 = -0.34172, \nu_4 = -0.30000, \nu_5 = -0.12316 + j0.17760, \nu_6 = -0.12316 - j0.17760$ and $\nu_7 = -0.09318$.

The tie-line resistance values are taken as $R_{12} = 2.0, R_{13} = 6.0, R_{14} = 9.0, R_{15} = 4.0, R_{23} = 2.0, R_{24} = 5.0, R_{25} = 8.0, R_{34} = 2.0, R_{35} = 3.0$ and $R_{45} = 2.0\,\Omega$.

The maximum generation limit for all areas and the maximum transmission capacities for all tie-lines are taken as 1500 MW and 600 MW respectively. We assume that transmission voltages and power factors are 380 kV and 0.9 for all tie-lines in throughout the system.

Initial loads of area-1 through area-5 are selected as 400.0, 400.0, 600.0, 900.0 and 1000.0 MW respectively. Two load changes are given to the system. In the first one, load changes for area-1 through area-5 are taken as 1.0, 3.0, 5.0, 7.0 and 9.0 MW respectively. In the second one, load changes for area-1 through area-5 are chosen as $-10.0, -7.0, -4.0$, 2.0 and 10.0 MW respectively. Negative load change means load decrease in the above. We will give the same initial loads, the first and the second load changes to the system in the following two cases for the comparison purpose.

Case 1

We have not made any changes in the numbers given in the above for the current case. The steady state results of the current case are given in Table 1. With the given load values, there is no generation and tie-line constraint hit in the system. We see from Table 1 that area-1 is an absolute seller area, area-2 is a net seller area, area-3 and area-4 are a net buyer areas, and area-5 is an absolute buyer area in the system. Note that the posted spot price of area-1 is equal to the base spot price of the system. Change of Δf and P_{m1} are shown as case-1 curves (solid lines) in Figure 2 and 3 respectively. Change of P_{m2} throught P_{m5} resembles the change of P_{m1}, but having different steady state values.

Case 2

We have taken tie-line 14 and tie-line 35 out of the system. We also have taken $P_{m1_{max}} = 1100MW$. The steady state results are given in Table 2. Since the generation of area-1 (the most efficient area in the system) hits its upper generation limit, the other areas in the system increase their generations. Taking out the tie-line 14 from the system causes area-4 to increase its bought power over the other tie-lines which are tie-line 24 and tie-line 34. Taking out the tie-line 35 from the system has the same effect on area-5 (see Table 1 and Table 2). Change of $\Delta f, P_{m1}$ are shown as case-2 curves (dash dotted lines) in Figure 2 and 3 respectively. P_{m1} curve overlaps with time axis for case-2 in Figure 3. Change of SP, SP_1 through SP_5 for case-2 is shown in Figure 4.

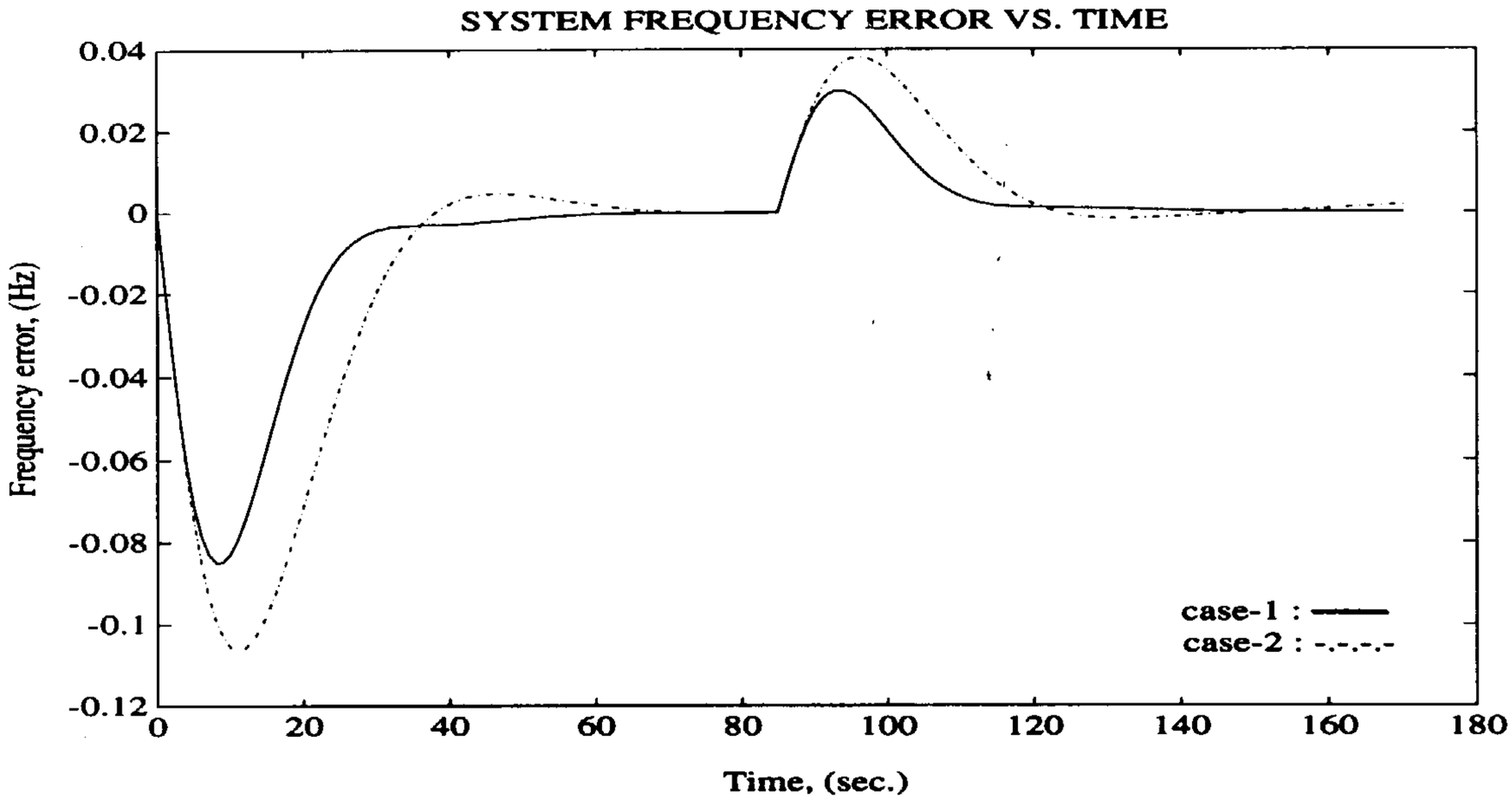


Figure 2. Change in the system's frequency error.

Table 1. Steady state values for area generations, frequency and time error, spot prices and tie-line transmission powers and loss power in case-1.

Steady state quantities	Initial	First	Second
$P_{m1}(MW)$	1140.51901	1149.11975	1146.06363
$P_{m2}(MW)$	755.80232	761.57685	759.54418
$P_{m3}(MW)$	574.14132	578.51049	577.00104
$P_{m4}(MW)$	458.78263	462.30060	461.10842
$P_{m5}(MW)$	380.92439	383.86222	382.88303
$\Delta f(Hz)$	0.00000	0.00005	0.00001
$\Delta Time(sec.)$	0.00000	-2.27991E-2	-1.47188E2
$SP_1(\not{R}/MWh)$	115.05190	115.91190	115.60638
$SP_2(\not{R}/MWh)$	115.37034	116.23664	115.93164
$SP_3(\not{R}/MWh)$	115.82826	116.70205	116.40022
$SP_4(\not{R}/MWh)$	116.19565	117.07510	116.77710
$SP_5(\not{R}/MWh)$	116.27731	117.15861	116.86493
$SP(\not{R}/MWh)$	115.03738	115.89741	115.59209
$TPT_{12}(MW)$	161.98523	163.85956	164.66037
$TPT_{21}(MW)$	-162.43358	-164.31835	-165.12365
$TPT_{13}(MW)$	131.63780	132.97459	133.95736
$TPT_{31}(MW)$	-132.52608	-133.88101	-134.87723
$TPT_{14}(MW)$	129.28779	130.50553	131.70241
$TPT_{41}(MW)$	-130.57307	-131.81514	-133.03615
$TPT_{15}(MW)$	311.66675	314.71846	318.55920
$TPT_{51}(MW)$	-314.98631	-318.10335	-322.02721
$TPT_{23}(MW)$	232.28472	234.39960	236.55014
$TPT_{32}(MW)$	-233.20668	-235.33842	-237.50627
$TPT_{24}(MW)$	167.46022	168.88993	170.72145
$TPT_{42}(MW)$	-168.65816	-170.10841	-171.96650
$TPT_{25}(MW)$	115.01858	116.06727	117.78411
$TPT_{52}(MW)$	-115.92278	-116.98804	-118.73232
$TPT_{34}(MW)$	185.62914	187.06235	189.50233
$TPT_{43}(MW)$	-186.21793	-187.66027	-190.11595
$TPT_{35}(MW)$	151.25943	152.62592	155.77112
$TPT_{53}(MW)$	-151.84584	-153.22298	-156.39305
$TPT_{45}(MW)$	41.13082	41.72735	44.00209
$TPT_{45}(MW)$	-41.15973	-41.75710	-44.03518
$P_{loss}(MW)$	10.16969	10.36250	10.60291

7. Discussion and Conclusion

We proposed a power dispatch technique for multi-area interconnected systems in this paper. The basic idea behind the dispatch technique is to introduce a spot price of electricity in the determination of the optimum generation of areas and the optimum power transfer between areas in the system. We used a base spot price

of electricity which is linearly dependent upon the system frequency error and the integral of the system frequency error. As long as the system loss is ignored, using the base spot price in the algorithm is enough to find the optimum generation of each areas. Using this base spot price of electricity makes the areas in the system behave more independently to maximize their own benefits. This kind of behavior of areas in the system lessens the necessary communication among them.

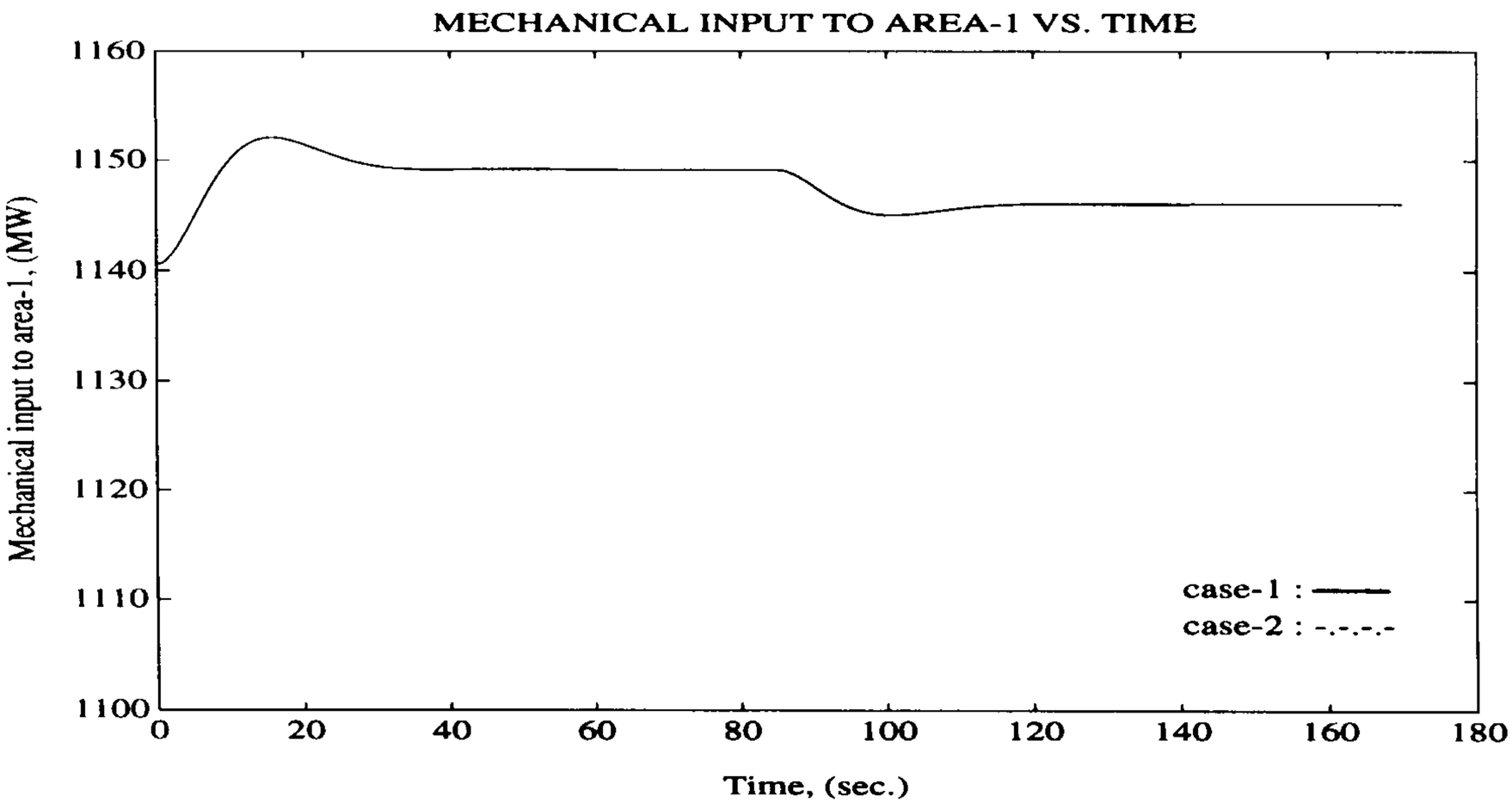


Figure 3. Change in the mechanical input to area-1.

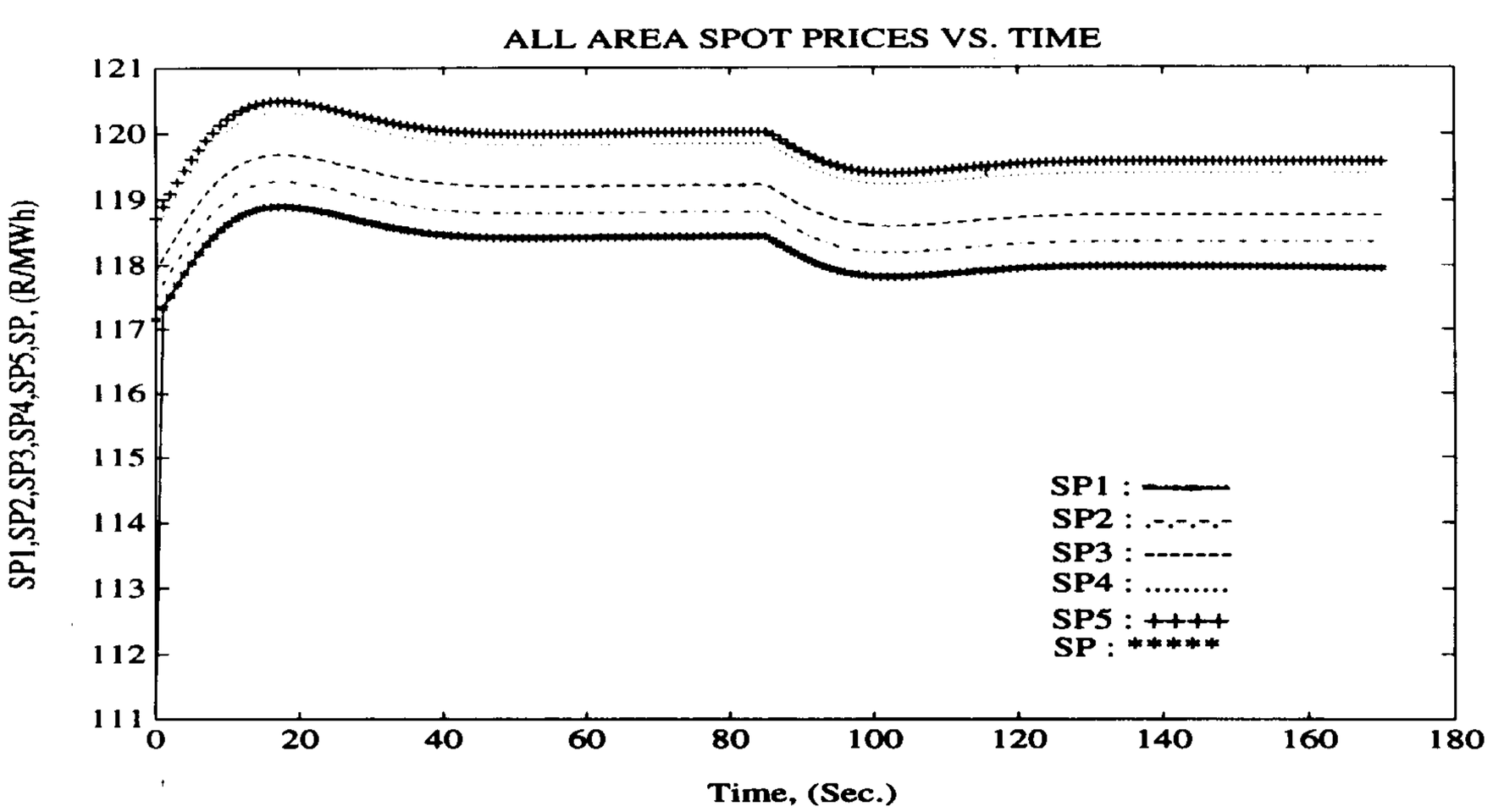


Figure 4. Change in all of the spot prices in case-2.

Table 2. Steady state values for area generations, frequency and time error, spot prices and tie-line transmission powers and loss power in case-2.

Steady state quantities	Initial	First	Second
$P_{m1}(MW)$	1100.00000	1100.00000	1100.0000
$P_{m2}(MW)$	770.20814	778.76577	775.75256
$P_{m3}(MW)$	584.64622	591.09666	588.84458
$P_{m4}(MW)$	468.19990	473.40507	471.63057
$P_{m5}(MW)$	389.03349	393.38869	391.93062
$\Delta f(Hz)$	0.00000	0.00018	0.0018
$\Delta Time(sec.)$	0.00000	-0.03400	-0.02149
$SP_1(\not{R}/MWh)$	111.00000	118.45170	117.95943
$SP_2(\not{R}/MWh)$	117.53122	118.81516	118.36290
$SP_3(\not{R}/MWh)$	117.92924	119.21948	118.76893
$SP_4(\not{R}/MWh)$	118.54997	119.85146	119.40776
$SP_5(\not{R}/MWh)$	118.71004	120.01699	119.57920
$SP(\not{R}/MWh)$	117.15703	118.45170	117.94890
$TPT_{12}(MW)$	181.56588	179.52534	181.99990
$TPT_{21}(MW)$	-182.12918	-180.07605	-182.56589
$TPT_{13}(MW)$	126.79057	126.44688	127.79434
$TPT_{31}(MW)$	-127.61464	-127.26649	-128.63151
$TPT_{14}(MW)$	0.00000	0.00000	0.00000
$TPT_{41}(MW)$	0.00000	0.00000	0.00000
$TPT_{15}(MW)$	385.18594	386.68235	392.62977
$TPT_{51}(MW)$	-390.25633	-391.79221	-397.89802
$TPT_{23}(MW)$	198.20474	199.23082	200.78455
$TPT_{32}(MW)$	-198.87602	-199.90906	-201.47341
$TPT_{24}(MW)$	202.91675	204.20562	206.63880
$TPT_{42}(MW)$	-204.67567	-205.98695	-208.46284
$TPT_{25}(MW)$	146.74970	148.01251	150.35383
$TPT_{52}(MW)$	-148.22162	-149.50987	-151.89894
$TPT_{34}(MW)$	308.02113	310.23531	314.80635
$TPT_{43}(MW)$	-309.64231	-311.87989	-316.49974
$TPT_{35}(MW)$	0.00000	0.00000	0.00000
$TPT_{53}(MW)$	0.00000	0.00000	0.00000
$TPT_{45}(MW)$	79.03076	80.83026	84.02486
$TPT_{54}(MW)$	-79.13748	-80.94190	-84.14549
$P_{loss}(MW)$	12.08777	12.19332	12.5434

When the system loss is included in the dispatch technique, the transfer powers between areas become important. We define the spot price of electricity for every area by using the base spot price or the price of power that is purchased. At this point, knowing the rules (or algorithm) for determination of the transfer powers between areas becomes important. Therefore, we proposed a transfer power algorithm and demonstrated it on the most general five area problem. According to this power transfer algorithm, a buyer area's spot price is found by increasing the spot price of the seller area by an amount that is equal to

the percentage of the loss that occurred in buying of that power. Actually during a power transfer between a seller and a buyer area, the buyer area receives an amount of power which is less than the power that is send by the seller area by the amount of the power lost during the transmission. Therefore the buyer areas sees a higher spot price of electricity than the spot price of the seller area.

We assumed that in every area the generating units can be represented as one generating unit, their input output curves can be put together as a one input output curve. One algorithm to do this job is given in [15].

Our dispatch algorithm does not handle the situation where one of the areas in the system is isolated. The most important problem in one area isolation event is to make sure that the system remains stable after the isolation of one area occurs in the system. This can be insured by choosing appropriate A_2 and A_3 coefficients for the base spot price of the system. The selection procedure of A_2 and A_3 coefficients can be as follows:

1. A specific value is assumed for A_3 constant.
2. The interval of A_2 values which makes the system stable when no area is isolated is determined.
3. The interval of A_2 values which makes the system stable when area-1 in the system is isolated
4. The same calculation given in item 3 is done when area-2 through area-5 is in isolation one at a time in the system.
5. At the end, we will have six intervals for A_2. The subinterval which is common in all six interval is determined. A suitable A_2 value which resides in the common interval is chosen.
6. If a common subinterval is not found, a new A_3 value is chosen and the procedure given between item 1 and item 5 is repeated.

A_2 and A_3 values obtained in this way will ensure the system to stay stable after one of the areas in the system gets isolated.

In the proposed dispatch technique, every area only needs to know the posted spot prices in areas to which it is connected. Communication is necessary amoung the areas that are connected for learning the posted spot prices and in power purchase arbitration.

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Çok Bölgeli Interkonnekte Sistemler İçin Spot Elecktrik Fiyatını Kullanan Bir Optimal Dağıtım Tekniği

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Özet

Bu çalışmada çok bölgeli interkonnekte sistemler için spot elektrik fiyatını kullanan bir optimal dağıtım tekniği önerilmektedir. Çok bölgeli sistemde bulunan bölgeler sabit fiyatlı elektrik alım satım anlaşması yerine önceden elektriğin taban spot fiyatının formülasyonu üzerinde anlaşmaktadırlar. Bu anlaşma bölgeler arasında zamanla değişen bir alım satım programı sonucunu doğurmaktadır. Önerilen taban spot elektrik fiyatı sistemdeki frekans hatası ve bu hatanın integralinin doğrusal bir fonksiyonu şeklindedir. Sistemde bölgeler arası hat kayıpları ihmal edildiği müddetçe, bölgelerdeki optimal güç üretimlerinin bulunmasında sadece taban spot elektrik fiyatının kullanılması yeterli olmaktadır.

Önerilen optimal dağıtım algoritması kayıplı çok bölgeli şekle genişletilmiş ve bu genişletme işlemi beş bölgeli interkonnekte sistem üzerinde gösterilmiştir. Bölgeler arası hat kayıplarının probleme dahil edilmesi taban spot elektrik fiyatının genişletilmesiyle başarılmıştır. Verilen transfer edilebilir bölge güçleri için bölgeler arası optimal güç transferi algoritması geliştirilmiştir. Sözkonusu transfer algoritması sistemde bölgeler arasındaki bazı hatların çıkarılabildiği değişik sistem konfigürasyonlarını, üretim ve bölgeler arasındaki hat kısıtlarını da gözönüne alabilmektedir. Dağıtım tekniği bölgelerdeki en optimal üretimlerle birlikte bölgeler arasındaki en ekonomik güç alış verişlerini vermektedir. Önerilen dağıtım tekniği ekonomik dağıtım yanında sistemdeki frekans regülasyonunu da sağlamaktadır.