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1pEAa8. Scattering of sound waves at an area expansion in a cylindrical flow duct

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Sound propagation in pipes and ducts with flow, like ventilation ducts and exhaust pipes, is influenced by flow separation and vortex production at sharp edges along the ducts, such as at bends and area expansions. Shear layers form at the separation points, and such layers are unstable to low frequency acoustic disturbances. An analytical model, aiming at physical insight into this interaction is presented. Results in the plane wave region for the so called scattering matrix for a sudden area expansion with flow in cylindrical pipes are compared with experimental values. Both the magnitude and the phase, in the form of an end correction, is presented. The model is also compared to a 2 dimensional model, in order to evaluate the anticipated increased accuracy of the 3 dimensional modeling. The scattering coefficients are strongly dependent on the flow speed, which is up to a Mach number of 0.5. It is observed that for low frequencies, the interaction is dominated by the dynamics of an unstable shear layer downstream of the edges. For higher frequencies, the wave propagation is mainly affected by convective effects. Differences in properties for the 2D and the 3D case are also explored.

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INTRODUCTION

Modeling of acoustic wave propagation in duct and pipe systems with flow is an important part in the prediction of radiated noise and choice of relevant noise abatement. At sections in the flow duct system where flow separation and vortex shedding occurs, the flow acoustic interaction may have a significant influence on the propagation properties. As the need for efficient numerical simulations increase, it is of interest to investigate how these phenomena can be simulated such that the significant effects of the flow on the acoustics are included. One way to investigate this is through the use of analytical models that can point to the influence of different parameters involved.

A number of models have been proposed for the scattering of sound waves at a sudden are expansion. A review of the early works is found in [1]. For a two dimensional (2D) model for a rectangular geometry, models have recently been proposed by, e.g., Aurégan [2], Boij and Nilsson [1, 3, 4], and Kierkegaard et al [5, 6]. Results have been compared with experimental data found in [7] obtained for a cylindrical geometry. A frequency normalization proposed by Boij and Nilsson [1] is used to make the comparison for the two different geometries possible. The correspondence has been rather good, despite the inherent differences in geometry. The influence of hydrodynamic coupling, flow profile and flow expansion downstream of the expansion has been studied.

In this paper, we extend the analysis and present a three dimensional (3D) analytical model for scattering of plane acoustic waves incident on a sudden area expansion in a flow duct with a cylindrical geometry. One of the purposes is to investigate how much the geometry influences the results, and to what extent the inclusion of 3D effects improves the results. The scattering matrix for the area expansion is calculated with the 3D model for cylindrical geometry. The results are compared both with experimental data and with results for the scattering matrix obtained with the 2D model for rectangular geometry proposed by Boij and Nilsson [1,3].

THE MODEL

In order to obtain a model that predicts the scattering properties of a flow duct area expansion, the proposed analytical modeling consists of two major parts. First, the linear wave equation is solved in each of the sections of the duct, i.e., one section with a uniform mean flow, and one with a mean flow profile which has a core with uniform flow speed (corresponding to the continuation of the smaller duct) that is surrounded by a region of quiescent flow, as depicted in Figure 1. Together with boundary and coupling conditions, it provides a modal solution describing the wave propagation in each duct. For the scattering of plane waves at the junction of the small and the large ducts, the modal excitation is determined by Wiener-Hopf methods. The details of the modeling follow the same principles as the modeling proposed earlier by Boij and Nilsson [1] and is briefly described here. The main difference is the new geometry such that the wave equation is here solved for cylindrical ducts.

For the formulation of the wave propagation problem, a flow model is chosen. In the narrow duct, the flow is assumed constant over the cross section. The flow profile in the wider duct is chosen such that the flow from the upstream duct continues undisturbed, as indicated in Figure 1, and is surrounded by a region of quiescent fluid. A discontinuity is thus introduced in the flow profile, and the field is determined applying coupling conditions requiring continuity in pressure and normal displacement across the jump, as well as rigid walls. The modes in the narrow duct are change by the convection when compared with duct modes with no flow. In the wider duct, the solution will contain the acoustic modes together with two hydrodynamic modes propagating with a speed proportional to the mean flow speed. One of these hydrodynamic waves is growing exponentially downstream of the expansion, while the other is decaying, both waves propagating for all frequencies. As the shear layer downstream of the expansion is modeled as infinitely thin, with the jump in mean flow profile, the shear layer remains unstable for all frequencies. This instability is the reason for the existence of the growing hydrodynamic mode. In reality, the shear layer is unstable for low frequencies, but becomes stable at a certain critical frequency.

In order to solve the scattering problem, an edge condition has to be applied at the corner of the area expansion. This edge corresponds to the point where the flow separation occurs. The edge condition thus models the behavior of the entire flow field in the vicinity of the sharp edge at the duct area expansion. Here, the Kutta condition is applied. The scattering problem is first solved for a so called bifurcated duct configuration, with two concentric ducts of which the smaller is semi-infinite and carries a mean flow. A building block method is then employed to arrive at the geometry of an area expansion, as described in [1].

In this paper we are interested in determining the scattering matrix, S, defined as

$$\begin{bmatrix} p_1^+ \\ p_2^+ \end{bmatrix} = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} \begin{bmatrix} p_1^- \\ p_2^- \end{bmatrix}, \tag{1}$$

where the pressure amplitudes with a '-' sign corresponds to incident waves travelling towards the area expansion, and the '+' sign corresponds to excited waves travelling away from the area expansion. The elements in the scattering matrix correspond to the scattering coefficients for acoustic waves in the plane wave region. Through the procedures used to model the scattering coefficients, the results presented here include the effect of the excitation of the hydrodynamic modes in the larger duct.

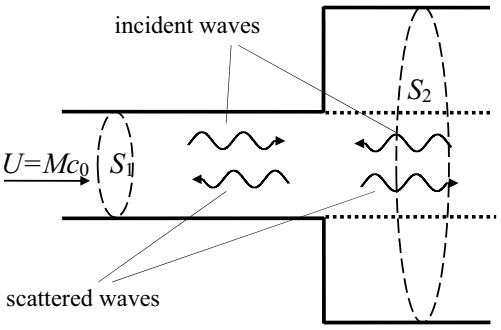


Figure 1. Geometry in the experiments and in the 3D model for cylindrical geometry.

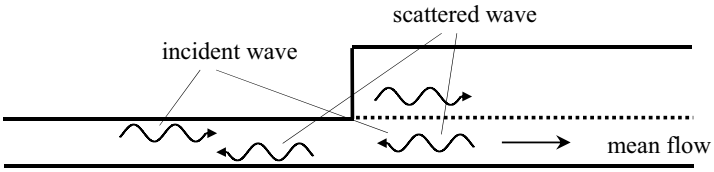


Figure 2. Geometry in the 2D model for rectangular geometry

Results calculated with the 2D model for rectangular geometries, presented in [1] are also presented; the geometry is sketched in Figure 2. The frequency normalization, based on the cut on frequency on the first higher order mode in the larger duct, as proposed in that paper is adopted.

RESULTS

Now, results for the scattering matrix, S, are presented for different frequencies, f, for a constant Mach number, M, of 0.29 for the mean flow in the narrow duct. The area expansion ratio is 0.346.

Comparison with Experimental Data

In Figure 3 a)-d) the magnitude of the scattering coefficients in S are shown, calculated with the 3D model for cylindrical geometry together with experimental results. The results are shown as a function of the Strouhal number, defined as $St = ka/M$, where $k = 2\pi f/c$, with c being the speed of sound, and a the radius of the narrow duct. The numerical values for the radius a is 0.025 m and for the frequencies 500, 1000, 2000, 2500 and 3000 Hz. Note that the scales on the y-axis are different in the different graphs. Overall, the correspondence is good. We can observe that the correspondence is better for the coefficients representing scattering into the wider duct, which are s_{21}

(transmission from the narrow duct) and s_{22} (reflection). Also, these two coefficients vary much less with the Strouhal number than the other two, which represent scattering upstream into the narrow duct.

In Figure 4 a) – d), the phase of the elements in S are shown in a similar way. We see that the trend is the same in experiments as in analytic data for all elements except s_{21}. However, it should be noted that the experimental data for the phase are not as accurate due to a larger relative error in the data given. The end corresponding to the reflection in the narrow part, defined as in [4] is shown in Figure 5. Here, we see that the correspondence is not so good, in particular at low Strouhal numbers.

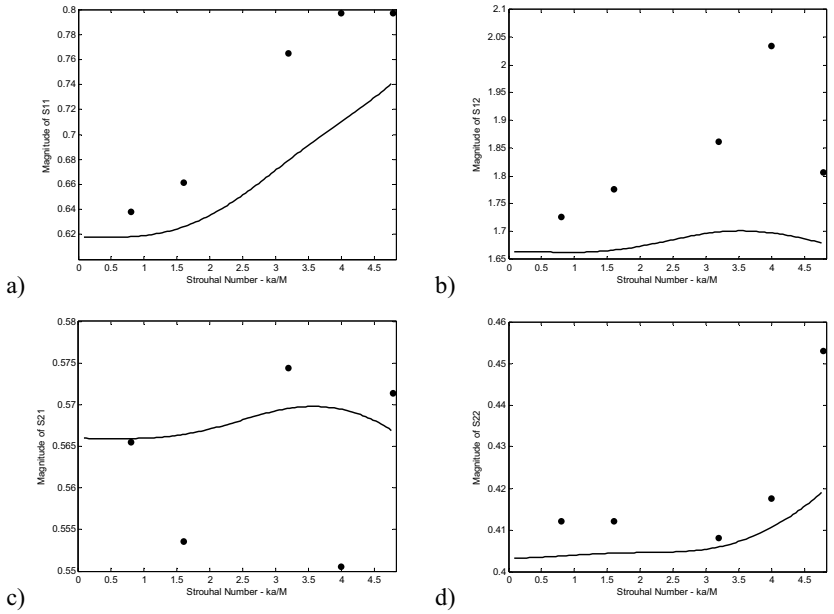


Figure 3 The magnitude of the elements of the scattering coefficients for Mach number in the mean flow of 0.29 in the narrow duct. Experiments [7]: dots, analytical model: solid line.

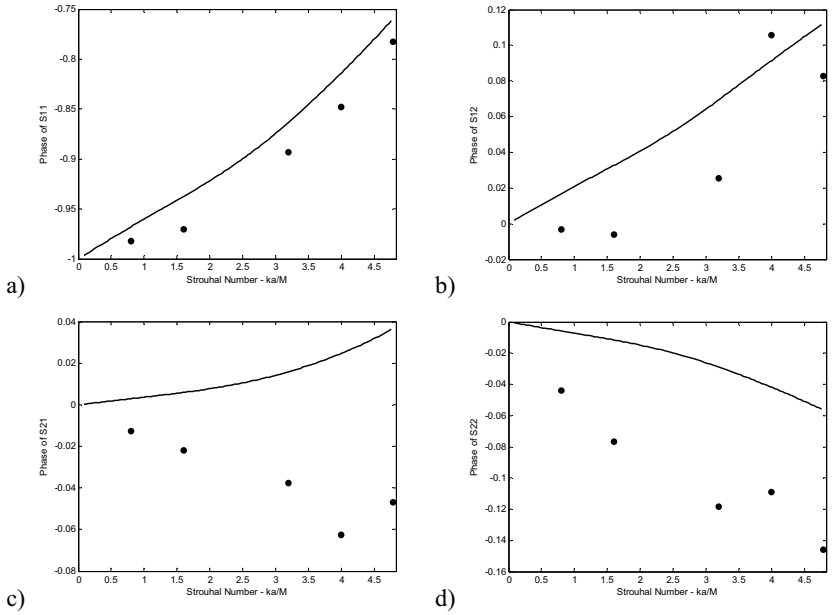


Figure 4 The phase of the elements of the scattering coefficients, normalized by π, for Mach number in the mean flow of 0.29 in the narrow duct. Experiments [7]: dots, analytical model: solid line.

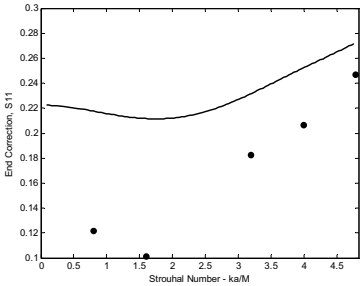


Figure 5 The end correction corresponding to elements s_{11}, for Mach number in the mean flow of 0.29 in the narrow duct. Experiments [7]: dots, analytical model: solid line.

Comparison between 3D cylindrical and 2D rectangular models

In this section, graphs showing results for the scattering matrix calculated with the 3D cylindrical and 2D rectangular models, respectively, are presented. In order to compare the data for the two geometries, the frequency normalization proposed by Boij and Nilsson [1] is applied. It should be noted that the Strouhal number on the *x*-axis is based on the *ka* for the cylindrical geometry. The area ratio is kept the same in the two models.

First the magnitude of the coefficients is shown in Figure 6 a)-d). We observe that for small Strouhal numbers, the two models give the same values. This result is expected as the details of the geometry should have no influence in the low frequency limit. At a Strouhal number of 2, the two models differ the most. Above this, it is assumed that the geometry starts to introduce differences between the models, due to the differences in geometry. The phase is then shown in the graphs of Figure 7 a)-d. The correspondence is very good. The discrepancies follow the same pattern as for the magnitude, i.e. the two models give the same solution in the low frequency limit, increasing difference up to Strouhal number 2, and higher Strouhal numbers, the difference in phase is constant in the Strouhal number regime considered here. However, when the end correction is considered, shown in Figure 8, the difference is rather pronounced between the models.

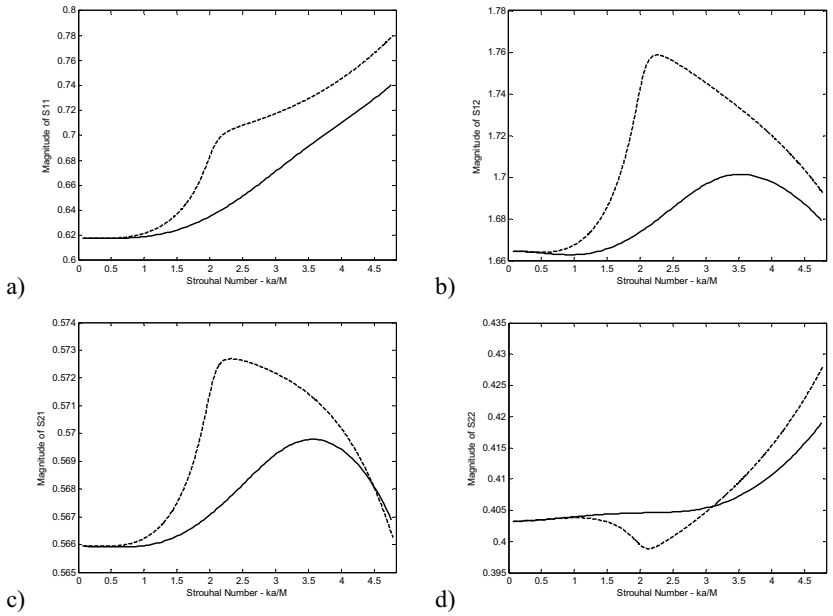


Figure 6 The magnitude of the elements of the scattering coefficients for Mach number in the mean flow of 0.29 in the narrow duct. 3D model for cylindrical geometry: solid line, 2D model for rectangular geometry: dashed line.

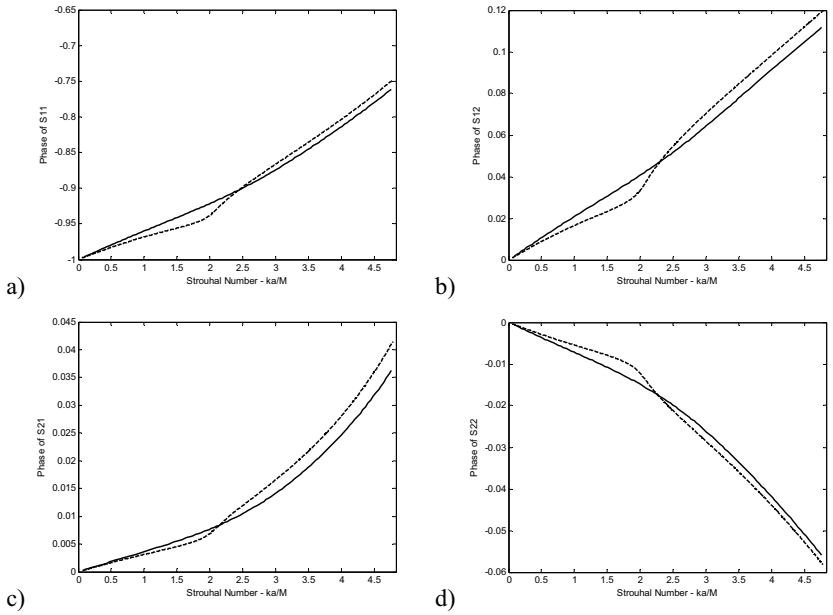


Figure 7 The phase of the elements of the scattering coefficients for Mach number in the mean flow of 0.29 in the narrow duct. 3D model for cylindrical geometry: solid line, 2D model for rectangular geometry: dashed line.

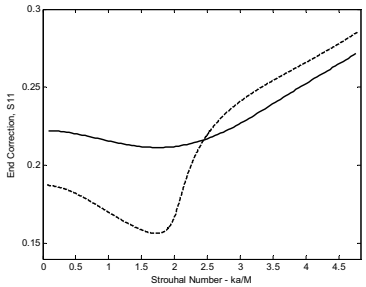


Figure 8 The end correction corresponding to elements s_{11}, for Mach number in the mean flow of 0.29 in the narrow duct. 3D model for cylindrical geometry: solid line, 2D model for rectangular geometry: dashed line

SUMMARY AND CONCLUSIONS

An analytical model for sound propagation in flow ducts is presented. A three dimensional model for cylindrical geometry with a mean flow from the narrow duct is presented. The results are compared with experimental data and also with values calculated with a 2D rectangular analytical model derived by the same principles.

The results show that the 3D model gives a good prediction of the scattering coefficients for plane waves incident from upstream and downstream, respectively. Rather surprisingly, however, the 3D cylindrical model does not improve the results achieved by the 2D rectangular model.

The results indicate that the actual geometry of the ducts is not critical in the modeling in the parameter regime under study. Also, the approximation introduced by the simplification of the geometry in the 2D model accompanied by the frequency scaling gives rather reliable results.

A preliminary study of the wave numbers for higher order acoustic modes and the hydrodynamic modes suggests that the flow acoustic interaction becomes less pronounced with the 3 cylindrical model. Thus, in an effort to improve the modeling, the results imply that the next step should be to investigate the modeling of the dynamics of the shear layer.

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